University of Alberta

Interpretation and Implementation of the Process of Communication in an Elementary Mathematics Classroom

by

Monica Anne Ellis

A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfillment of the requirements for the degree of Doctor of Philosophy

Department of Elementary Education

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled **Interpretation and Implementation of the Process of Communication in an Elementary Mathematics Classroom** submitted by Monica Anne Ellis in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

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Abstract

The purpose of this study was to explore one Grade 6 teacher's interpretation and implementation of the process of communication as identified in the *Common Curriculum Framework for K-9 Mathematics: Western Canadian Protocol for Collaboration in Basic Education Program of Studies* (Alberta Education, 1996). Recent instructional reforms within mathematics education have proposed dramatic shifts in the way mathematics is conceptualized. It is now conceptualized within a constructivist framework and includes an emphasis on student talk and writing as essential to meaning making in mathematics. The emphasis on talk and writing in mathematics has been translated as the process of communication in many mathematics documents.

The design of the study was situated within the context of a qualitative case study. The participants were a Grade 6 teacher and her class of 23 students. Data-collection procedures included the writing of reflective and descriptive field notes, participant observation, interviews with the teacher, and the collection of documents. The data were analyzed by identifying common themes that emerged within and across data sources.

The results of the study indicated that contextual variables had a powerful influence on the teacher's interpretation and implementation of the process of communication in mathematics. The culture of the classroom and school where Terry and the students engaged in routines and accepted protocols, and where time constraints always loomed, were integral to the types of decisions Terry made. Provincial tests, the nature of the school subjects, the subject-specific orientations towards constructivism and the term *communication*, the WCP document, and the *Quest 2000* textbook also had a great impact on Terry's instructional decisions. Terry's beliefs about constructivism,

classroom environments that best facilitate learning, and the need for accountability were also critical components affecting how she interpreted and implemented the process of communication.

As a result of the contextual variables, the teacher's practice in relation to the implementation of the process of communication in mathematics fluctuated along a continuum ranging from transmission (teacher-directed) to interpretation (student-oriented). The contextual variables tended to support a transmission model of teaching and a teacher-directed orientation to the process of communication.

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CHAPTER 1

INTRODUCTION

In 1997 the *Common Curriculum Framework for K-9 Mathematics: Western Canadian Protocol for Collaboration in Basic Education Program of Studies* (Alberta Education, 1996) became the prescribed mathematics curriculum for the province of Alberta. New to this mathematics curriculum document was a constructivist orientation, including a focus on talking and writing in mathematics. The mathematical process of communication was listed as one of the seven processes critical to a mathematics program in order to achieve the goals of mathematics education. As a doctoral student in language arts and a former elementary school teacher, I wanted to explore what the implementation of the process of communication looked like in an elementary mathematics classroom and what this could tell educators about the link between curriculum and classroom practice. I had always been interested in how talking and writing assist in student learning. My master's thesis explored how dialectic journals were used in a Grade 3 social studies classroom. I wanted to learn more about aspects of communication in the teaching of mathematics. I conducted this study, with the assistance of a Grade 6 teacher, in order to explore how the process of communication was interpreted and practiced in a Grade 6 mathematics classroom.

My own teaching history in relation to language arts and mathematics is probably typical of that of many elementary teachers. School had not interested me much when I was young, but I found myself increasingly attracted to working with children. When I decided to become a teacher in the early 1980s, I envisioned having a classroom in which children were happy and free to explore, a contrast to my life as a student in the 1960s.

As a teacher I was able to accomplish my goal of creating an active, student-oriented classroom; for example, in social studies and science I ensured that students were extensively involved in using journals, learning logs, report writing, debates, and discussion. I believed, and still do, that language is our tool for making sense of the world and that students need many opportunities for using oral language, writing, and reading in the classroom.

Mathematics is a subject that, I must admit, perplexed me as a teacher. As a student I had been accustomed to teacher-directed instruction. My mathematics learning followed the same pattern year after year. We watched the teacher teach a math concept on the chalkboard, and then we worked quietly at our desks. I remember lining up at the teacher's desk to have my work corrected. The only discussion was the forbidden talk that happened while we waited in this line. As a teacher, I found it difficult to reconcile the teacher-directed instruction I utilized in mathematics with the hands-on, interactive approach I favored in other subjects. Part of me was comfortable with the transmission style of instruction, and another part of me knew that interaction and language were extremely important in mathematics learning. When I look back, I believe I was in a constant state of turmoil as to how to teach mathematics. I knew that children had to talk and write about their mathematics learning, but I could not reach a comfort zone in my own classroom. I took in-service workshops on problem solving in mathematics and tried to implement the ideas in my own classroom. I wanted my students to be able to talk and write about their mathematics and to problem-solve together, but I was unsure of how to facilitate these strategies.

Although I thought of myself as a constructivist teacher, believing that students construct meaning through experience and interaction with others, I did not always apply this to my teaching of mathematics. I somehow believed that mathematics was different; made up of absolutes, rights and wrongs, with no room for discovery and exploration. I knew that talking and writing helped children learn in subjects such as science and social studies. My experiences with mathematics and mathematics instruction did not allow me to translate my beliefs about how children use language into my teaching of mathematics.

Background to the Study

The increased focus on the process of communication in mathematics has developed within mathematics education in the last 13 years. Reys, Suydam, Lindquist, and Smith (1998) state, "The importance of communication in mathematics learning is demonstrated by the fact that communication is one of only four National Council of Teachers of Mathematics (NCTM) standards that are highlighted at all grade levels" (p. 24; see Appendix A.) In 1989 both Canada and the United States initiated mathematics reform, calling for the importance of mathematical literacy (NCTM, 1989, 1991; National Research Council [NRC], 1989). The National Council of Teachers of Mathematics, the foremost North American organization for mathematics education, declared in 1989 that one of the major goals of school mathematics should be to increase the amount and quality of children's mathematical communication. The shift from an industrial to an information society has seen an increased emphasis on communication in all areas of life. The NCTM's (1989) *Curriculum and Evaluation Standards for School Mathematics* document states, "Information is the new capital and the new material, and communication is the new means of production" (p. 3). The *Common Curriculum*

Framework for K-9 Mathematics: Western Canadian Protocol for Collaboration in Basic Education document (Alberta Education, 1996) provides Alberta's interpretation of the NCTM standards. (It will be referred to from now on as the WCP document.) Communication is incorporated into this document as one of seven processes listed as critical to a mathematics education program (see Appendix B). Because the WCP conceptualization of communication is central to this research study, the following statements about communication from the WCP document are presented below:

> Students need to communicate mathematical ideas clearly and effectively, orally and in writing. Communication will help students make connections among different representations of mathematical ideas; namely, "physical, pictorial, graphic, symbolic, verbal and mental representations." (NCTM, 1989, p. 26)

> It is not enough to arrive at an answer. Students must be able to communicate effectively how the answer was obtained. In other words, students need opportunities to read, to explore, to investigate, to write, to listen to, to discuss and to explain ideas in their own language of mathematics. Thus, students can create their own links "between their informal, intuitive notions and the abstract language and symbolism of mathematics." (NCTM, p. 26; WCP, p. 6)

These were the statements regarding the process of communication available to the teacher and me at the time of the research.

Educators such as Cobb, Yackel, and Wood (1992a), Cohen and Ball (1990), Schifter (1996a), and Simon (1995) state that there is a need for teachers to know, to make sense of, and to carry out the standards set out by the NCTM. They stress that teacher educators and educational researchers must listen to teacher voices with respect to educational reform and that teachers must be the primary agents of the process. Schifter (1996a) states, "But once the urgency of the need for teacher dialogue has been understood, the absence of teacher voices from the on-going national conversation about mathematics education reform becomes deafening" (p. 79). Hiebert (1999) comments on

the importance of the relationship between research and the NCTM Standards, stating, "Research can provide information about how we are doing at the moment, how we are teaching, what curriculum materials we are using, and how students are learning. Although this is an obvious role for research, it is often underutilized" (p. 8). Schifter observes that we know little about how teachers are implementing the NCTM standards in mathematics classrooms in Canada and the United States.

Teachers' experiences are integral to the examination of the facilitation and implementation of the new reforms. Their participation in research is vital. Teachers have to be able to discuss their experiences in carrying out the standards of the mathematics curriculum, including the process of communication. Healy and Barr (1991) stress the need to dialogue about language across the curriculum practices, saying, "The necessary collaboration among teachers of different subjects, as they put language across the curriculum practices to work, opens classroom doors, relieving the isolation of teaching" (p. 825). Teachers need a forum for sharing what they are learning as they follow the new standards.

As I began this research study I believed that it was important to find out how one teacher was interpreting the process of communication, as identified by the WCP document (Alberta Education, 1996), in her mathematics classroom. I believed that this would lend voice to the dialogue with respect to educational reform within the mathematics field and would inform our understandings about teachers and their implementation of prescribed curriculum. I hoped to add to the body of research on teacher change and language across the curriculum practices.

Purpose of the Study

Recent instructional reforms within mathematics education have proposed dramatic shifts in the way mathematics is conceptualized. The WCP document (Alberta Education, 1996) was mandated for use in Alberta schools in the fall of 1997. The focus on the process of communication is a North American trend as evidenced in the NCTM's (1989) *Curriculum and Evaluation Standards for School Mathematics* document. To respond to these reforms requires that many teachers change their practices. However, we know little about how Alberta teachers are conceptualizing and enacting the process of communication within the teaching of mathematics.

The purpose of this study was to examine and document how one Grade 6 teacher constructed meaning about the process of communication in a Grade 6 mathematics classroom.

Research Question

The research question guiding the study was, "How does one teacher interpret and put into practice the process of communication as identified in the WCP document?" The focus of the study was on how one teacher made sense of the emphasis on the process of communication within the subject of mathematics.

Theoretical Framework

I chose the constructivist paradigm as the theoretical framework for my study because it provides a focus on meaning making for the research and is the theoretical stance embedded in the process of communication as conceptualized in the WCP. Constructivism is a philosophical position about the nature of knowledge and reality. This philosophical position views knowledge as a human construction: Humans do not

find or discover knowledge as much as they make or construct it. Knowledge is seen as temporary, internally constructed, and socially and culturally mediated. Much of the modern-day discussion about constructivism can be traced to the ideas of Lev Vygotsky, a Russian psychologist who conducted research within the 1920s and 1930s. He was interested in how children conceptualize the meanings of words and concluded that social interaction and communication are essential components in this conceptualization process (Vygotsky, 1994). Within the theoretical paradigm of constructivism are many interpretations. All share the idea that our understandings of the world come from the point of view of those who live in it (Schwandt, 1998). Schwandt states:

> The world of lived reality and situation-specific meanings that constitute the general object of investigation is thought to be constructed by social actors. That is, particular actors, in particular places, at particular times, fashion meaning out of events and phenomena through prolonged, complex processes of social interaction involving history, language, and action. (p. 222)

The focus of my research was the interpretation and facilitation of a curriculum document which becomes a socially mediated event in the classroom. The teacher and students strive to make sense of curriculum through their interactions with each other. The process is not static; it is always evolving. The classroom researcher also becomes a meaning maker within this community of learners.

Significance

The findings of this study provide significant information for teachers, researchers, and curriculum developers who are interested in how practitioners interpret aspects of a newly mandated mathematics curriculum. This study adds voice to the research literature in mathematics education, particularly in reference to the interpretation of the process of communication as stated in the WCP document (Alberta Education,

1996). The study offers insights into how the constructivist orientation is interpreted and implemented and examines the many contextual influences affecting a teacher's instructional decisions. It also contributes to our understandings of how opportunities for student talking and writing assist in the learning of mathematics. Presenting how the WCP document was interpreted by one teacher and her students may encourage curriculum developers and mathematics educators to re-examine how new curricula are introduced and implemented and as a result encourage closer examination of teacher in-servicing and preparation in order to determine what is needed to ensure success for both teachers and students.

Overview of the Study

Chapter 2 presents a review of the literature relevant to the study. I profile constructivist theory, social constructivist theory, constructivism as interpreted in the fields of language arts and mathematics and the debates within these fields, teacher knowledge paradigms, and a review of some of the relevant studies done in the area of mathematics reform. Chapter 3 presents the research framework and the research procedures that include (a) selection of the teacher, (b) data-collection methodologies, and (c) data-analysis techniques. In Chapter 4 I present the contextual considerations that were so important to this study. Chapter 5 presents the findings with respect to the interpretation and implementation of the process of communication as it was facilitated in the Analyzing Data and Graphs Unit. Chapter 6 presents the findings with respect to the interpretation and implementation of the process of communication from the Measuring and Analyzing Angles Unit, the Exploring Geometric Solids Unit, and the Measuring

Area and Perimeter Unit. In Chapter 7, I present the review of the findings, conclusions, implications, and recommendations for future research.

CHAPTER 2

LITERATURE REVIEW

Introduction

To establish a theoretical framework for this study I initially examine constructivism and social constructivist theory as philosophical frameworks. I examine how constructivism is generally interpreted in the language arts field because this is my area of expertise and crucial to an examination of the process of communication in an area such as mathematics. The concept of language across the curriculum is discussed as a context for the focus on the process of communication in the subject area of mathematics. Theoretical shifts within mathematics education and how constructivism is defined within the field of mathematics are then discussed. The concept of communication is explored, along with its theoretical constructs within mathematics. I profile some of the relevant research on teacher knowledge, beliefs, and context because these became important reference points for coming to an understanding of the complexities of teacher decision making. I conclude by describing the relevant research conducted in the area of teachers' implementation of the mathematics reforms.

Introduction to Constructivist Theory

Constructivism is a philosophical position that views knowledge as a human construction (Gredler, 2001). Constructivist theory asserts that humans do not find or discover knowledge so much as they construct or make it. Knowledge is seen as temporary, internally constructed, and socially and culturally mediated. Within the theoretical paradigm of constructivism there are many interpretations. Schwandt (1994) states, "All share the idea that our understandings of the world come from the point of

view of those who live in it" (p. 118). Major differences within the field center on discussions related to Piagetian and Vygotskian views on constructivism.

Fosnot (1996) mentions that it is Piaget's work completed in the 10 to 15 years prior to his death that serves as the psychological basis of constructivism. Piaget at this point in his career focused on the process that enabled new constructions to come about. Piaget believed that the human was a developing organism, not only in the physical and biological sense, but also in the cognitive sense. He talked about the roles of "contradiction" and "equilibration" in learning. When a child experiences a "contradiction," something that is unpredicted and not working in the way the child anticipated, then this causes an imbalance, providing internal motivation for accommodations. *Equilibration* is defined as a dynamic process of self-regulated behavior balancing two intrinsic polar behaviors, assimilation and accommodation (Fosnot, 1996). Knowledge is realized through successive constructions on the part of the individual.

Piaget's work is generally considered to center on the cognitive structuring of individuals, but he acknowledged the role of social interaction in learning. Piaget (1970) writes, "There is no longer any need to choose between the primacy of the social or that of the intellect; the collective intellect is the social equilibrium resulting from the interplay of the operations that enter into all cooperation" (p. 114).

The basic premise of Vygotsky's theory of constructivism is that the cultural context and other symbols shape a child's view of reality. These symbols also serve as psychological tools that are important in developing higher cognitive functions such as self-regulated attention and conceptual thinking. Because the source of knowledge is the

continuing interactions and reciprocity between a developing child and a simultaneously changing world, Moshman (1992) refers to Vygotsky's perspective as dialectical constructivism.

The social constructivist stance from the Vygotskian tradition best defines my standpoint on constructivism. Social constructivists emphasize Vygotsky's crucial point that knowledge is constructed by cultures. Gergen and Gergen (1991) explain:

> Accounts of the world . . . take place within shared systems of intelligibility—usually spoken or written language. These accounts are not viewed as the external expression of the speaker's internal processes (such as cognition, intention), but as an expression of relationships among persons. (p. 78)

Collective generation of meaning is shaped by language and social interactions. Social constructivists from a Vgotskian tradition view the classroom as a community charged with the task of developing knowledge. Knowledge is seen as inseparable from the activities that produced it (Bredo, 1994; Dewey & Bentley, 1949). As an elementary teacher for 13 years, I realized very early in my career that the classroom was comprised of a community of learners, including me. Nothing was learned in isolation within this setting; we were all part of the same sustained community of practice.

It has been necessary for me to examine the emergence of constructivist thought in the fields of both mathematics and language arts because their theoretical orientations are somewhat different.

Constructivist Theory In Language Arts

How constructivist theory is defined in language arts is important as a philosophical base for understanding the process of communication in mathematics. Language arts curriculum development was heavily influenced by constructivist theory in

the 1980s and 1990s. Understandings about how children learn and what they need for learning to take place have been greatly influenced, in my view, by a few key people. Lev Vygotsky (1962) and Jerome Bruner (1986) have figured prominently in language arts educators' understandings about the importance of language and social interaction for extending and developing human learning. Their work influenced my practice in the elementary classroom; opportunities for talk and social engagement became part of the everyday climate. No matter what subject, my students were always involved in conversations and group work. In the context of curriculum, the work of Vygotsky and Bruner has been invaluable to understandings of the necessity of language use through all subject areas.

Social constructivists such as Barnes (1992) assert that children construct meaning in the world through the use of language as a tool of exploration. The research and theories of Vygotsky (1962) have contributed greatly to our knowledge about the nature of language and thought in the field of language arts and other subject areas. Vygotsky's pioneering work focused on language as promoting and accompanying the growth of thought. Vygotsky argues that thought and language develop simultaneously, that one does not exist without the other. Language is considered the verbal mediator that organizes and clarifies thought. He believes that the development of language enables higher-order thought to take place. This relationship does not remain static. Vygotsky emphasizes that the relationship between language and thought undergoes many changes and that progress in thought and speech are not parallel.

Vygotsky believes in the social nature of learning and is interested in dialogue. He believes that the primary function of speech was communication (social intercourse) and

that verbal interactions between children and adults help children learn how to mean. He is interested not only in the role of inner speech, but also in the role of the learner's peers as they help a child come to understandings by conversing, questioning, explaining, and negotiating meaning (Fosnot, 1996). Vygotsky (1986) describes the way the verbalization of ideas and feelings often leads to understanding them better, saying, "Language serves as the deliberate structuring of meaning" (p. 100). This, combined with social interaction, leads to thought. When children can talk about their worlds and interact socially with others, it helps them to negotiate thought. Vygotsky writes, "Thought development is determined by language; i.e., by the linguistic tools of thought and by the sociocultural experience of the child" (p. 94). It is the effects of social interaction, language, and culture that are the cornerstones of a child's learning and growth.

Jerome Bruner has greatly influenced language arts educators' understandings about language and thought. Bruner (1986) explores the development of language and its precursors. He wants to know what it is that readies the child so early for transacting his life with others on the basis of workable intuitions about other minds and human situations. Bruner argues that language and culture are powerful tools in allowing transactions to take place. In the act of "transaction" we come to understand ourselves and the cultural group to which we belong. Individuals change as a result of their engagement with each other; the known and the knower are both changed. According to Bruner, "Learning how to use language involves both learning the culture and learning how to express intentions in congruence with culture" (p. 65). Bruner believes that most of our approaches to the world are mediated through negotiation with others and that cognition, affect, and actions work together as a unified whole to create a social being.

Bruner (1964) believes that language allows us to think about events that have happened in our lives and select from them significant lessons for use in the present. He writes:

> Once language becomes a medium for the translation of experience, there is a progressive release from immediacy. For language . . . has the new and powerful features of remoteness and arbitrariness: it permits productive combinatorial operations in the absence of what is presented. (p. 14)

Through language we are able to reflect on our experiences in life and make sense of it all.

Bruner (1975) proposes the notion of "scaffolding," which initially centers on the communication rituals between mothers and infants in which a mother and child jointly construct meaning through their turn-taking dialogue. He believes that it is the modeling and direct instruction that gives children the tools to guide them in their next steps.

Vygotsky's and Bruner's theories have proven instrumental to me in my understandings of children's learning. Their theories reflect the belief that for learning to take place, children need many opportunities in the classroom for the verbalization of ideas and feelings, coupled with many opportunities for social engagement. Students and teachers are constantly constructing and reconstructing their worlds in the classroom. Language is a crucial part of the learning process. Bruner's and Vygotsky's theories provide us with a social constructivist perspective in the field of language arts. However, as in the mathematics field (Cobb, 1996), there is debate in the language arts field around the nature of constructivism.

Language Across the Curriculum

Just as Bruner's and Vygotsky's theories figure prominently in current notions of constructivist thought, their theories help formulate the intellectual basis for language across the curriculum. The idea that knowledge is the product of a complex interaction between people, their experiences, and language is the cornerstone of the language across the curriculum movement. The phrase *language across the curriculum* has become synonymous since the 1960s with a focus on the role of students' oral and written language in the learning of a range of subjects in school. Healy and Barr (1991) write:

> The principles underlying language across the curriculum in schools emphasize the necessary link between language and thought, build on the prior experiences all pupils bring to school, and acknowledge the learner's intentions to make sense of new information, including subject matter concepts. (p. 820)

The conceptualization of language across the curriculum grew out of the meetings and dialogue created by the London Association for the Teaching of English. There were a few key educators from Britain whose work greatly influenced understandings of language across the curriculum. In 1966 a group of secondary English teachers met to consider the role of talk in English lessons. Harold Rosen, Mike Torbe, James Britton, and Douglas Barnes were key people who worked on this. Parker (1985) writes that they found it difficult to confine their study to English lessons alone and invited colleagues from other subject areas to join them in their work. Barnes, Britton, and Rosen (1971) explain:

> We found ourselves talking about 'language in education,' or 'language and learning,' and finally about language across the curriculum. We felt sure that language was a matter of concern for everyone, that if children were to make sense of their school experience, and in the process to become confident users of language, then we needed to engage in a much closer scrutiny of the way in which they encountered and used language throughout the school day. (p. 1)

Many publications emerged as a result of discussions within this group and with other teachers. *Language Across the Curriculum: Guidelines for Schools* (Torbe, 1976) is a pamphlet intended to give practical suggestions for developing language policies in one's school. *Language, The Learner, and the School* (Barnes et al., 1971) was published and became a catalyst for change in the field of language arts. It discusses teacher-pupil dialogue, talk as a way of learning, and the implications of a whole-school policy for language. There was subsequently another edition of this book. In the 1986 edition Barnes, in his chapter, "Language in the Secondary Classroom," does an analysis of teacher-talk that demonstrates the way in which a teacher could *prevent* learning rather than generate it by the way he or she talks to pupils and allows them to respond.

In 1976 Barnes was the sole author of a book called *From Communication to Curriculum*, which is devoted to the importance of communication in the curriculum. He writes about language entering the curriculum as the communication system of the classroom and school and as a means of learning. Barnes argues:

> A communication system is an abstraction from the behavior of a group of people; from one point of view it constrains them, but from another it exists only through what they do. The meanings that we live by change because we insensibly day by day renew them in the course of sharing our lives. This sharing is communication. (p. 31)

Barnes, like other constructivist educators, views language as providing a pivotal set of strategies for interpreting the world and as a means to reflect upon this interpretation, leading to the formulation of knowledge. He describes situations in classrooms that hinder children's formulation of knowledge, such as too much teacher-directed talk, lack of opportunities for children to engage in talk, and teachers' evaluative comments in response to children's talk. Barnes proposes an interactive model of teaching and learning

that focuses on the pupils. He stresses the need to look at a curriculum theory that includes a social model that acknowledges both learner and a social milieu and includes communication from pupil to teacher as well as teacher to pupil. Barnes suggests that the way teachers think about what constitutes knowledge is often linked to what they think teaching and learning are. He points out that their view of knowledge is likely to carry with it a view of classroom communication, which in turn plays out in the classroom.

Barnes (1992) proposes a continuum (Figure 1) based on the extremes of transmission and interpretation. He initially used this continuum to plot 11th-grade teachers' attitudes towards their students' written work. According to Barnes, the interpretation teacher perceives the teacher's task to be the setting up of a dialogue in which the learner can reshape his or her knowledge through interaction with others. The interpretation teacher believes that knowledge exists in the knower's ability to organize thought and action. The transmission teacher believes that knowledge exists in the form of public disciplines, which include content and criteria of performance and perceives the teacher's task to be the evaluation and correction of the learner's performance, according to criteria of which he is the guardian. According to Barnes, if a teacher sees knowledge as existing primarily in the knower's ability to interpret, he will emphasize the reply aspect of his classroom role, making possible a negotiation between his knowledge and his pupils' knowledge. Barnes states, "This will open to them a collaborative approach in which the exploratory functions of speech and writing predominate" (p. 147).

Transmission		Interpretation
Recording		Cognitive Development
	(purpose)	
Acquisition of information		Personal Development
Product		Context
	(awareness)	
Task		Pupil's attitude
Assessment		Replies and comments
No follow-up	(responses)	Future teaching
Corrections		Publish

Figure 1. Transmission-interpretation model (Barnes, 1992).

James Britton is a member of the London group, and his work has figured prominently in the field of language arts. Britton (1970) synthesizes Bruner's and Vygotsky's theories and urges that schools build on the inclination of human learners to use their talk, their reading, and their writing for real-life purposes. He talks about the interrelatedness of language arts, "the marriage of the process of composing in written language to that of reading, and the relating of both to the learner's spoken language resources" (p. 159). Britton is a firm believer in the social, interactive view of learning. In a 1971 study of the group talk of 16-year-old girls published in *Language, the Learner, and the School*, Britton found their talk much richer when it was not sanctioned by a classroom teacher. As a result of this study, and others he has done, he is convinced that "the network of people related to us by shared values provides, at every stage of life, the primary context for our learning of both varieties—coming to know and refining our

value systems" (p. 106). Britton sees expressive talk and writing as a means to negotiate meaning in the world. *Expressive language* is the term given to someone's own familiar language; it is the voice closest to us and the one we feel the most comfortable using because it is tied to the context of what we are doing (Britton, 1970). Since Britton views talk as an integral means of learning, he believes that the onus for establishing an atmosphere in which students feel free to engage in talk lies with the classroom teacher. His views assume an interactive view of learning and emphasize the importance of opportunities for collaboration and student interaction in curriculum documents and classroom practice.

Mike Torbe (1981, 1986) and Peter Medway (1980) are also key members of the London group and have spent a great deal of time studying the talk and language going on in classrooms. They write extensively about the importance of talk and its counterpart, writing, across the curriculum. They emphasize that because we learn by talking and interaction, then student talk is a crucial part of the classroom. Students' writing offers a glimpse into the ways that students are learning. Torbe (1981) writes about the need to include language, talk, and writing as a major part of learning in any subject and says that it is through our use of these that we would indeed find out what our students are truly learning. He writes:

> A science teacher, another English teacher, a sociology teacher, a math teacher—all of them had a common concern for their pupils' learning. They didn't begin by being concerned with language, but found that listening to the pupils' talk, looking at their writing, and considering their reading as well as thinking increasingly carefully about what they themselves were saying—these offered insights into the ways students were learning. (p. 8)

Medway (1980) is convinced of the importance of language in all subjects, not just in the subject we call English. He states, "Language is, or ought to be, the concern of all of

them [subjects]; all promote learning and knowing-and not just 'knowing how to' but knowledge about the world" (p. 7). Medway describes the process his college went through when they combined the humanities: English, geography, religious education, and social studies. The students are given choice, self-monitoring activities, and lots of opportunities for writing and talking. What results is a rich experience for students and teachers. Examination results are higher, and the students are happier.

Both Torbe and Medway are firm believers in the necessity of teachers' encouraging expressive language, both spoken and written, in the classroom. They believe that expressive language, throughout the process of learning, becomes the ally of understanding because it makes explicit the lessons of experiences.

As a result of the discussions and publications of the members of the London Association for the Teaching of English, the Bullock (1975) report, *A Language for Life*, was published. It has become a national call for schoolwide, agreed-upon language policies in England. Its central premise is that learning takes place primarily through language. Bullock states:

> To bring knowledge into being is a formulating process, and language is its ordinary means, whether in speaking or writing or the inner monologue of thought. Once it is understood that talking and writing are means to learning, those more obvious truths that we learn also from other people by listening and reading will take on a fuller meaning and fall into proper perspective. (p. 50)

The Bullock report provides guidelines and a rationale for implementing language across the curriculum. It asserts that every school should have a policy for language across the curriculum. One of the recommendations in the report is, "Each school should have an organized policy for language across the curriculum, establishing every teacher's involvement in language and reading development throughout the years of schooling"

(p. 514). The Bullock report states that there are important inferences to be drawn from a study of the relationship between language and learning: "All genuine learning involves discovery, and it's ridiculous to suppose that teaching begins and ends with 'instruction' as it is to suppose that 'learning by discovery' means leaving children to their own resources" (p. 50).

The work completed by the London group and subsequent articles and publications have become pivotal references for language arts educators' practices in Canada, England, and Australia (Parker, 1985). Their work has had a great deal of significance in terms of its emphasis on students as meaning makers and the relationship between language and learning.

Constructivism in Mathematics

The constructivist perspective on learning has become central to much of the recent theoretical and empirical work in mathematics education (Steffe & Gale, 1995; von Glasersfeld, 1995). But what is meant by the term *constructivism* in mathematics varies. Cobb (1996) writes, "There is currently a dispute over whether the mind is located in the head or in the individual-in-social-action, and whether learning is primarily a process of cognitive reorganization or a process of enculturation into a community of practice" (p. 35). He goes on to say that the issue of whether social and cultural processes have primacy over individual processes is a subject of intense debate.

Collectivism/Individualism

Paul Cobb, Gotz Krummheuer, Jorg Voigt, Terry Wood, Erna Yackel, and Heinrich Bauersfeld constitute a group of American and German mathematics educators whose research has had a profound impact on the changing theoretical perspectives in the field of mathematics education in the last 14 years.

Cobb, Yackel, Wood, and Simon have been instrumental in the creation of a combined psychological/sociological perspective as a way to understanding how mathematics learning occurs. As a result of three years of conversations in which they discussed varying ideas about how students learn in mathematics, Simon, Cobb, and Yackel have created a combined psychological/sociological perspective that acknowledges the importance of an individual student's mathematical interpretations while simultaneously seeing activity as socially situated (Cobb, 1995; E. Yackel, personal communication, December 1998).

The terms *collectivism* and *individualism* (Cobb & Bauersfeld, 1995) are found in the discussions of the psychological and sociological perspectives. The *collectivist* philosophy is derived from theories developed in the Vygotskian tradition. Studies conducted by Carraher, Carraher, and Schliemann (1985), Lave (1988), Saxe (1991), and Scribner (1984) have supported this tradition in mathematics. These researchers contend that an individual's mathematical activity is profoundly influenced by his or her participation in encompassing cultural practices such as completing worksheets in school, shopping in a supermarket, selling candy on the street, and packing crates in a dairy (Cobb & Bauersfeld, 1995). These mathematics theorists adhere to the Vygotskian contention that "the social dimension of consciousness is primary in fact and time. The

individual dimension of consciousness is derivative and secondary" (p. 4). Learning is considered to be located in co-participation in social processes. The *individualism* philosophy treats mathematical learning almost exclusively as a process of active individual construction. This position has been exemplified by neo-Piagetian theories which view social interaction as a source of cognitive conflicts that facilitate autonomous cognitive development (Cobb & Bauersfeld, 1995).

Emergence of Social Constructivist Theory in Mathematics

The terms *constructivism* and *social constructivism* have been emphasized in the last 10 years by some mathematics researchers, but there is a plethora of different meanings given to these terms (Steffe & Gale, 1995). Social constructivism in mathematics has been conceptualized more recently in the work by mathematics researchers and educators such as Cobb, Yackel, and Wood (1992a, 1995), Simon (1995), and von Glasersfeld (1991, 1995). They have combined their varying philosophies to come up with a perspective of social constructivism that includes Piagetian and Vygotskian ideas. This is an emergent perspective that permits analyses of learning from both the social (group) and the psychological (individual) perspectives. The core idea is that learners actively construct their own understandings rather than passively absorb or copy the understandings of others. The construction of new understandings is stimulated by a problem situation that disturbs the individual's current organization of knowledge. This disturbance occurs when an individual's current cognitive structures do not adequately solve, explain, or predict what is happening. Disequilibrium leads to mental activity and a modification of previously held ideas (Simon & Schifter, 1991). The social becomes internalized as individual.

A significant part of this theory, emphasized by Cobb, Yackel, and Wood (1992, 1995) and Ernest (1991), is the importance of the knowledge shared by the community. An individual's cognitive structure cannot be understood without observing its interacting in a context, within a culture. Wood et al. (1995) assert, "It is useful to see mathematics as both cognitive activity constrained by social and cultural processes, and as a social and cultural phenomenon that is constituted by a community of actively cognizing individuals" (p. 402). This view shifts the role of the teacher from imparting a set of "truths" to helping connect the students' mathematics with the mathematics of the larger community. Teaching mathematics has been reconceived as the provision of activities designed to encourage and facilitate the constructive process. Schifter (1996) states:

> Individuals necessarily approach novel situations by interpreting them in the light of their own established structures of understanding; that the construction of new concepts is provoked when those settled understandings do not allow satisfactory accommodation of the novel circumstance; and that this constructive activity is not simply an individual achievement but is embedded in and enabled by contexts of social interaction. (p. 78)

Shifter's explanation of how children 'come to know' in a mathematics classroom is very much reflective of a Vygotskian approach to constructivism. According to Cobb (1996), the debate continues with respect to whether "the mind is located in the head or in the individual-in-social-action" (p. 35); however, within a social constructivist framework the role of the community in coming to know mathematics is crucial.

Classroom Culture and Communication In Mathematics

Pragmatic studies of communication in relation to mathematical learning have been rare (Bauersfeld, 1995). Ideas about communication in mathematics have been changing with the theoretical changes concerning this subject in the last decade. In a

mathematics classroom, how we use communication and theoretically look at communication has a profound effect on what happens in that classroom. For many years the representational view in mathematics has had a strong hold in mathematics education. Language was thought of as the vehicle through which one merely relates what one has found or the answer one has come up with. Bauersfeld (1995) states, "This way of looking at communication in mathematics comes from the Aristotelian view that sees language as the third (objective) issue between the two givers—the subject and object" (p. 277).

There has been a slow trend on the part of mathematics educators to view communication as language arts educators have collectively done: communication as a social event, socially mediated and constructed. Bauersfeld (1995) writes about the importance of the processes of communicating in mathematics and their impact on personal development, rather than with language as an object or as a given medium that can be used. Relying heavily on Wittgenstein (1974), Bauersfeld prefers to speak of "languaging" instead of the connotations of language use such as "speech" or "speaking." Drawing on Vygotsky (1931) and Bruner (1984), Bauersfeld refers to "understanding" as "the active construction of . . . meanings and references supported by the social interaction within the culture" (p. 274).

Notions of communication in mathematics are enveloped in the culture of the mathematics classroom, as they are within culture as a whole. Within the mathematics field the social 'taken-as-shared' activities give rise to the languaging that emerges across social interaction. Yackel and Cobb (1996), Cobb, Yackel, and Wood (1989), and Yackel, Cobb, and Wood (1991) have written extensively about the norms found in

mathematics classrooms. They write about these norms emerging, as they do in any culture, from taken-as-shared notions; for example, what counts as mathematically different, mathematically sophisticated, or mathematically efficient becomes important within the culture of the mathematics classroom. In this view of taken-as-shared norms, the development of an individual's reasoning and sense making cannot be separated from culture; in this case, the culture of the mathematics classroom and the culture of mathematizing as a practice. The idea that students are expected to explain their solutions and ways of thinking is a *social norm,* whereas the understanding of what counts as an acceptable mathematical explanation is a *socio-mathematical norm.*

It is within the culture of the mathematics classroom that the teachers' and students' socially mediated concepts of mathematics evolve. Language within the setting of the mathematics classroom plays a crucial role; it is the means by which we share and mediate our understandings of mathematics. Bauersfeld (1995) observes, "When language is (for the subject) 'an active mirror of experience' rather than 'a passive mirror of reality' or an adequate representation of it, then the qualities of the culture a person lives in become critical" (p. 283). Bruner (1986) speaks of language in the same realm:

> Language not only transmits, it creates or constitutes knowledge or 'reality.' Part of that reality is the stance that the language implies toward knowledge and reflection, and the generalized set of stances one negotiates creates in time a sense of one's self. (p. 133)

Bauersfeld (1995), Cobb (1989, 1995), and Voigt (1992) believe that both the students' mathematical activity and the social classroom micro culture have to be accounted for in any analysis of the mathematical classroom. It is not just enough to study the type of work in which the students are engaged, but an integral part of any study must also look at the culture of the classroom.

Vygotsky's and Bruner's theories provide theoretical support in understanding the concept of communication in mathematics. Bauersfeld (1995) in his discussion of communication in mathematics proposes a joint constructivist and interactionist alternative:

> If we had to identify common ground among the theories of Vygotsky, Piaget, and Bruner regarding the function and development of language, the answer would be the fundamental influence of social interaction. Children enter mathematics education at school with a more or less well developed mother tongue langúaging. What is new then? Mathematics opens a new perspective on the world. . . . What counts is the answer to "how many?" Further, dealing with the "how many" issues (numbers) develops into the exceptional situation (the math lesson) with special materials (representations), very special manipulations and signs (operations and number sentences), and a related special language game. It is a very strict game with strange rules about what is allowed and what is not, and the child's only access into this new subculture is active participation. (p. 284)

It is through the members' taken-as-shared activities that members agree on the same names and descriptions. The everyday practices of the members of the subculture of the mathematics classroom reproduce the culture as well as actively contribute to develop and change it. It is the negotiation of meaning in social interaction that functions as a starting point for an effective "language game" in the mathematics class.

Communication in mathematics represents an integral part of the classroom culture. There is no one way to study it, and, at best, researchers must at least try to define what aspects of 'mathematics communication' they wish to investigate.

Constructivism As Conceptualized by the National Council of Teachers of Mathematics

In 1989 the NCTM, the foremost North American organization of mathematics educators, reformed the standards for mathematics education in response to the call for reform in the teaching and learning of mathematics. A constructivist perspective informs the principles guiding the current movement for mathematics education (Schifter & Simon, 1991). One of the basic assumptions of the NCTM Curriculum and Evaluation Standards document is that the K-4 curriculum should actively involve children in doing mathematics. It states, "Young children are active individuals who construct, modify, and integrate ideas by interacting with the physical world, materials, and other children. Given these facts it is clear that the learning of mathematics must be an active process" (NCTM, 1989, p. 17).

The WCP document (Alberta Education, 1996) adopted the NCTM's philosophical position and described constructivism as, "Students learn by attaching meaning to what they do; and they must be able to construct their own meaning of mathematics" (p. 2).

Because constructivism has been viewed in the past in mathematics education as an epistemological theory instead of a definition of a particular way to teach, there is relatively little work in mathematics education that has focused on the development of a theoretical framework for mathematics pedagogy (models of teaching) consistent with constructivism (Shifter & Simon, 1991; Simon, 1995). Constructivism implies a new set of goals for the classroom but does not prescribe explicit instructional strategies. We

know little about how teachers create a teaching practice guided by constructivist principles in mathematics.

Constructivism has provided a basis for recent mathematics education reform efforts. The process of communication has become one of the crucial components of the reform efforts in mathematics. Several rationale statements were provided for this by the NCTM in the Curriculum and Evaluation Standards document: (a) There had been a shift from an industrial society to an information society; workers could no longer be illiterate; (b) people would change jobs four to five times in the next 25 years, and each job would require retraining in communication skills; and (c) society needed an informed electorate where citizens were able to read and interpret complex information.

Standard 2 of this NCTM document is titled, "Mathematics as Communication." It provides the rationale for communication in mathematics, emphasizing that young children are active, social individuals and that much of the sense they make of the world is derived from their communications with other people. The document states that communicating helps children to clarify their thinking and sharpen their understandings (NCTM, 1989). It suggests that mathematics include numerous opportunities for communication: "Representing, talking, listening, writing, and reading are key communication skills and should be viewed as integral parts of the mathematics curriculum" (p. 27). These are representative of a constructivist perspective. Pages 27-28 and 79-80 give brief examples of what the communication skills such as talking and writing might look like in elementary classrooms. Since the completion of my study, a more recent document, *Principles and Standards for School Mathematics* (NCTM, 2000), has been published. Communication in mathematics has been given more prominence

and elaboration than the previous Standards document. For example, one description of communicating through writing states:

> Writing in mathematics can help students consolidate their thinking because it requires them to reflect on their work and clarify their thoughts about the ideas developed in the lesson. . . . To support classroom discourse effectively, teachers must build a community in which students will feel free to express their ideas. . . . Starting in grades 3-5 students should gradually take more responsibility for participating in whole-class discussions and responding to one another directly. They should become better at listening, paraphrasing, questioning, and interpreting others' ideas. (p. 61)

Teacher Knowledge

Fenstermacher (1994) writes an extensive article on the nature of knowledge and research in teaching. He mentions the growing research literature on the knowledge that teachers generate as a result of their experience as teachers in contrast to the knowledge of teaching that is generated by those who specialize in research on teaching, which he labels "teacher knowledge: formal." Teachers actively construct knowledge just as students do. What teachers know as a result of their experience as teachers is what he refers to as "teacher knowledge: practical." These kinds of knowledge include practical, personal practical, situated, local, relational, and tacit. He groups Connelly and Clandinin (1988), Schon (1983), Shulman (1987), and Cochran-Smith and Lytle (1990, 1993) as scholars who are contributing to the literature in the category we call "teacher knowledge: practical."

Teacher Practical Knowledge/Professional Knowledge Landscape

Clandinin and Connelly (1988) have written extensively about "teacher practical knowledge." It is a term designed to "capture the idea of experience in a way that allows us to talk about teachers as knowledgeable and knowing persons" (p. 25). Teacher

practical knowledge is a particular way of reconstructing the past and the intentions of the future to deal with the exigencies of a present situation. Clandinin and Connelly discuss teacher practical knowledge within the context of the notion of "curriculum" as what is "lived" and "experienced" within the classroom by the teacher and the students.

In recent work, Clandinin and Connelly (1995) have written about the "professional knowledge landscape" that exists both inside and outside the classroom and is composed of relationships among people, places, and things and as an intellectual and moral landscape. They discuss the terms *theory* and *practice* as creating dilemmas in the lives of professional educators. They use the "conduit" metaphor as described by Reddy (1979) and Schwab (1970, 1971, 1973, 1983), where the "theoretical knowledge" is packaged for teachers in textbooks, curriculum materials, and professional-development workshops and transmitted via conduit to the teachers' professional knowledge landscape.

Teacher Knowledge in the Area of Mathematics

What a teacher knows is one of the most important influences on what is done in classrooms and, ultimately, on what students learn (Elbaz, 1983; Fennema & Franke, 1992; Shulman, 1985). Fennema and Franke outline knowledge of mathematics, knowledge of mathematical representations, teachers' knowledge of students, and teachers' general knowledge of teaching and decision making as important considerations in any discussions about teacher knowledge. They present a research model for examining the integration of teacher knowledge. Their model shows the interactive and dynamic nature of teacher knowledge and includes the components of teacher knowledge of the content of mathematics, knowledge of pedagogy, knowledge of students'

cognitions, and teachers' beliefs as working together to allow teachers' knowledge to develop in context. Fennema and Franke state, "Examination of teachers is beginning to indicate that knowledge can be transformed and is transformed through classroom interaction. When knowledge is transformed during instruction, that knowledge becomes tied to the context in which it was developed" (pp. 162-163).

Fennema and Franke's (1992) model is useful but not extensive enough to explain all the variables that are important when trying to understand decisions that teachers make about their practice. Their model does not thoroughly address the importance of context.

Relationship Between Beliefs About Mathematics and Instructional Practice

There are many studies in the mathematics field examining the relationships between reforms, beliefs, and practice (Brosnan, Edwards, & Erickson, 1994; Cohen & Ball, 1990; Prawat, 1992; Spillane, 2000). How a teacher interprets and implements curriculum can be related to beliefs about mathematics, but there are inconsistencies in the studies published in this area. Thompson (1984), Steinberg, Haymore, and Marks (1985), Grant (1984), and Shirk (1973) in their respective studies observe a high degree of consistency between teachers' beliefs about the nature of mathematics and instructional processes. Researchers such as Kesler (1985), Cooney (1985), Shaw (1989), and Thompson (1982) report some variability in the degree of consistency between teachers' conceptions and their teaching practice. Thompson (1992) alerts us to the fact that there are inconsistencies between professed beliefs and instructional practice and that any serious attempt to characterize a teacher's conception of the discipline should not be limited to an analysis of the teacher's professed views but should include an examination

of the instructional setting, the practices characteristic of that teacher, and the relationship between the teacher's professed views and actual practice. Prawatt (1992) suggests that not only teachers' beliefs about mathematics should be examined and challenged, but also teachers' views on the nature of knowledge. He writes:

> If teachers are to alter their teaching of mathematics, they may need to reexamine a whole network of beliefs extending far beyond their views about the craft of teaching, narrowly defined, they may need to change their views about the nature of knowledge and how one acquires that knowledge. (p. 211)

Context

The relationship between context and the teaching and learning of mathematics is important. Jones (1997) describes three faces of context: (a) the interpretation of policy, (b) school norms and expectations, and (c) conceptions of mathematics. Under the umbrella of interpretation of policy, Jones cites factors such as curricular reforms and local implementation as considerations. He cautions about top-down imposition of policy, saying:

> There is a substantial and well regarded body of analysis that supports the view that the difference between success and failure depends on local implementation and the degree to which local concerns fit or can be accommodated to those of the reform. (p. 135)

School norms and expectations are also a point of focus for studies of the influence of context on teachers' change. Schifter and Fosnot (1991), Bullough (1989), and Kidder (1989) have conducted studies that deal with particular classrooms, and their results point to the importance of the understanding of classroom dynamics. According to Ball (1988), Jones (1990), and Thompson (1984), how a teacher views mathematics has a significant impact on his or her classroom interactions and teaching goals. An important part of the

classroom dynamics is the role that students play in this. Shifter and Goldsmith (1997) and Jones (1997) write about students' understandings and attitudes towards math has having an impact on teacher change. Jones writes:

> A potentially tension-producing experience for teachers can be their observation of students' mathematical thinking. Under certain conditions, students have the opportunity to display their thinking, and teachers have reason to take note of it, not dismiss it. Because change in knowledge, beliefs, and practice occurs over time, it would be valuable to investigate the nature of the interaction between teachers' knowledge and beliefs, their growing skill at providing opportunities for students to demonstrate their thinking, and subsequent tensions. (p. 147)

In pursuing the literature on teacher knowledge and context, I came to the realization that there are many frameworks and variables to consider when trying to understand how a teacher makes decisions in mathematics. I found Jones's description of contextual features, provincial and school board policies with respect to mathematical instruction, school and classroom norms and expectations, and the teacher's conceptions of mathematics as crucial in coming to understand the choices that the teacher in my study was making. Clandinin and Connelly's (1995) notion of the professional knowledge landscape was also helpful in describing the teacher's practice.

Related Research in the Implementation of the NCTM Standards

There have been no studies conducted in Canada that have explored specifically how the mathematical process of communication from the WCP document (Alberta Education, 1996) is interpreted and put into practice in the elementary mathematics classroom, but there have been studies conducted in the U.S. that have explored teachers' implementation of the mathematics reform and related changes in instructional strategies and beliefs.

Richard Prawat (1992) conducts a case study examining a Grade 5 teacher's instructional strategies and views about mathematics teaching that occurred over the course of a year in response to California's movement to reform mathematics teaching. He finds that although there was an important change in the teacher's views about mathematics over the course of the year, this did not appear to be reflected in her classroom practice. Karen, the teacher, describes her approach to teaching as traditional and teacher directed, and her lessons in December reflect this. In March, Prawat observed that Karen was using more math manipulatives and had organized the students into cooperative groups, but he states, "On closer examination, however, this lesson, like the two described earlier, appears to contain more elements of traditional math than of the much more open-ended discourse-centered mathematics envisioned in the *California Mathematics Framework*" (p. 207).

Brosnan, Edwards, and Erickson (1994) conducted a two-year study to document and examine changes in four teachers' beliefs and practices while they implemented the NCTM standards. The participants are four elementary sixth-grade mathematics teachers. Qualitative research methods such as interviews, observations, journals, attitude and belief surveys, and videotapes were used to develop multiple case studies, which were analyzed individually and across cases. Problems reported include that the teachers had limited knowledge of (a) the NCTM's (1989) *Curriculum and Evaluation Standards for School Mathematics* document, (b) current mathematics teaching methodologies, and (c) mathematics content. Changes in the teachers' beliefs, which are reflected in their practice, include an increase in student-centered activities, the use of manipulatives and calculators, and good questioning techniques. The researchers conclude their study by

stating that teachers' most immediate need during the implementation of a new approach to the instruction of mathematics is information dealing with classroom practice and alternative assessment tools.

James Spillane (2000) conducted a comprehensive study that examined a teacher's instruction of language arts and mathematics. He conducted a four-year study commencing in 1991 that explored how a fifth-grade teacher taught mathematics and language arts as a result of reform movements in both of these subject areas in the 1980s. He studied Ms. Adams, a veteran teacher with 20 years' experience. Spillane finds different instructional practices in the subjects of mathematics and language arts. During mathematics the teacher established, without discussion, correct and incorrect answers. She maintained a tight rein on the discourse by posing all the questions, determining who answered her questions, and deciding legitimate issues for classroom discussion. With the exception of a small-group activity, students never initiated conversations, speaking only in response to questions posed by the teacher. Discourse norms in the mathematics lessons were especially influenced in defining mathematical knowledge and doing mathematics chiefly as a means of finding the correct algorithm. This is in sharp contrast to language arts, in which knowing is not just about memorizing facts and rules and applying them perfunctorily. There were many opportunities in her language arts lessons for student exploration of literature and discussion. Spillane concludes that subject-matter-sensitive differences in the teacher's identity as a teacher and learner influence very different learning opportunities, as well as substantial differences in her stance as a learner in mathematics compared with literacy. These differences in her learning

contributed to substantial variation in her enactment of reform in mathematics compared with literacy.

Research that has focused specifically on a teacher's conceptualization of the communication process in mathematics seems to be nonexistent, but there have been various case studies, such as those described above, that explore how teachers make sense of the new educational reforms. These studies helped to support my findings and shed light on the complexities involved in the implementation of the NCTM Standards.

Summary

Beliefs about how children learn have changed within the educational field in the last century. Psychologists such as Bruner and Vygotsky have informed current practices with respect to a social constructivist pedagogy in language arts. Their theories suggest that human beings are social beings and construct knowledge based on background experiences and interactions with others. *Language Across the Curriculum* (Torbe, 1976) has become an important concept within the language arts field in the 1960s and 1970s, changing instructional practices in many schools within Canada, England, and Australia.

Piaget (1970), Cobb, Yackel, and Wood (1992b, 1995), and Simon (1995) have informed constructivist theory in mathematics. *Social constructivism* has emerged as a term to describe a student's knowledge making within the realm of a social context and, most specifically, within the culture of the mathematics classroom. The locus of learning is not confined to the individual's mind but occurs in a community of participants and is distributed among co-participants. Knowledge is transactional, the result of interaction between or among individuals who together create new understandings. Learning is socially constructed and distributed among the co-participants.

The concept of communication in mathematics has emerged out of social constructivist research on thought and language, which emphasizes the link between language and knowledge and the importance of social engagement in constructing knowledge. An understanding of communication in mathematics has emerged from Vygotsky's and Bruner's theories. The NCTM's (1989) *Curriculum and Evaluation Standards for School Mathematics* document reflects constructivist understandings of how students learn and states that children should be actively involved in mathematics and have numerous opportunities for communication. Understandings of mathematics are shared and mediated through communication. Not only does language transmit, but it is also integral in the formulation of knowledge. There have been few pragmatic studies of communication in relation to mathematics.

Why and how teachers make the decisions they do about their practice is very complex. How teachers' knowledge and beliefs affect their practice and what comprises teacher knowledge are important to our understandings of teacher practice. The role of the professional landscape and the context of the mathematics classroom are important variables in our understandings of how teachers come to implement curriculum in the classroom.

There have been no studies to this point that have explored how teachers actually interpret and implement the process of communication as outlined in the WCP document (Alberta Education, 1996), but studies have looked at how teachers have implemented mathematical reforms in a general sense. These studies reflect that even though teachers may claim to view mathematics differently as a result of reforms, this is not always reflected in their practice.

CHAPTER 3

METHODOLOGY

Introduction

This chapter presents the qualitative research framework and a description of the research procedure, including (a) discussion of the constructivist paradigm in relation to research methodology; (b) discussion of case study; (c) finding Terry, the classroom teacher; (d) data-collection methodologies; and (e) data-analysis techniques and considerations of triangulation, transferability, limitations, and ethical considerations. I used a case study approach, conducted within a qualitative research paradigm, using interviews, observations, field notes, and documents as my data-collection methods.

Research Design Framework

I chose a qualitative research paradigm as a framework for my research because I was examining how a teacher interpreted curriculum in her classroom. Qualitative research is multimethodological in focus, involving an interpretive, naturalistic approach to its subject matter (Denzin & Lincoln, 1994, 1998). According to Merriam (1998), "Qualitative research is an umbrella concept covering several forms of inquiry that help us understand and explain the meaning of social phenomena with as little disruption of the natural setting as possible" (p. 5). Stake (1994) explains that the qualitative researcher pursues an understanding of the complex interrelationships among all the participants. The setting of a Grade 6 classroom provided a context for a rich qualitative inquiry.

Constructivist Paradigm

I used the constructivist paradigm as defined by Guba and Lincoln (1989) as the umbrella for my research. In the constructivist tradition realities are apprehended in the form of multiple, intangible mental constructions; are socially and experientially based; and are dependent for their form and content on the individual persons or groups holding the constructions (Guba & Lincoln, 1994). Guba and Lincoln look at the constructivist paradigm as being both idealistic and pluralistic. They assume that what is real is a construction in the minds of the individuals; in a pluralistic sense, there are multiple, often conflicting constructions, and all are potentially meaningful (Guba and Lincoln, 1985). Guba and Lincoln also assume that the observer cannot and should not be disentangled from the observed in the activity of inquiring into constructions. Schwandt (1998) points out, "The findings or outcomes of an inquiry are themselves a literal creation or construction of the inquiry process. Constructions in turn are resident in the minds of the individuals" (p. 243). Guba and Lincoln (1989) state, "They do not exist outside of the persons who create or hold them; they are not part of some objective world that exists apart from their constructions" (p. 143).

Constructivists are committed to the view that what we take to be objective knowledge and truth is the result of perspective and that knowledge and truth are created, not discovered by the mind. Research conducted in the constructivist paradigm seeks to find meaning in the particular research situation dictated by the human interaction that takes place in that realm. Bruner (1986) reminds us that constructivists endorse the claim that "contrary to common-sense there is no unique 'real world' that preexists and is independent of human mental activity and human symbolic language" (p. 95).

Discussion of Case Study

Under the umbrella of the constructivist paradigm I used a case study as the unit of study with which to carry out my inquiry. In Robert Stake's (1994) words, "Case study is not a methodological choice, but a choice of object to be studied. We choose to study the case" (p. 236). Stake (1998) argues that the concepts of *case* and the term *study* are open to different interpretations. Case study is the study of the particularity and complexity of a single case, coming to understand its activity within important circumstances. A case study is both the process of learning about the case and a product of our learning (Stake, 1998). Merriam (1988) defines case study as a detailed examination of one setting, or a single depository of documents on one particular event.

I chose an "intrinsic case study" as defined by Stake (1998). According to Stake, "Study is taken because one wants better understanding of this particular case. . . . Study is undertaken because of intrinsic interest in, for example, this particular child, clinic, conference, or curriculum" (p. 88). I went into this study curious about how one teacher interpreted and implemented the process of communication in one mathematics classroom, knowing that there would be no theory building.

Merriam (1988) states that case studies can be described by the nature of the final report. I engaged in an interpretive case study in which my data were used to develop conceptual categories. In an interpretive case study, a researcher gathers as much information about the problem as possible, with the intent of interpreting or theorizing about the phenomenon. Merriam suggests that the level of abstraction in case studies may range from suggesting relationships among variables to constructing theory. In my final analysis I suggested relationships among the variables. According to Stake (1998):

> Case researchers, as others, pass along to readers, some of their personal meanings of events and relationships-and fail to pass along others. They know the reader too will add and subtract, invent and shape-reconstructing the knowledge in ways that leave it differently connected and more likely to be personally useful. (p. 95)

I presented the results of my case study in the most objective way possible, always cognizant of the limitations inherent in my interpretations. Merriam (1988) asserts that case studies are limited by the "sensitivity and integrity of the investigator" (pp. 33-34).

The terms *case* and *story* have been used synonymously in recent literature on case study. In the last 10 years there has been growing evidence that cases and stories can offer rich data on how teachers come to understand their students, schools, communities, subject matter, and the overall practice of teaching (Carter, 1993; Miller 1990; Oyler, 1996; Schifter, 1996a; Shulman, 1992; Witherell & Noddings, 1991). Shulman states, "We are now strongly convinced that cases become catalysts for pedagogical conversations among members of school communities. They stimulate teachers' individual reflections on their own teaching as well as providing a basis for dialogue and interaction among teachers themselves" (p. xv). Case studies provide educators with the raw materials for their own generalizations and interpretations. Patton (1980) comments that the narrative genre of the case study makes it highly readable: "It makes accessible to the reader all the information necessary to understand...It takes the reader into the case situation, a person's life, a group's life, or a program's life" (p. 134). I hoped that through my presentation of one case of how a teacher interprets and implements the process of communication in mathematics, I would add one small contribution to the body of research on the implementation of the WCP (Alberta Education, 1996).

Research Procedure: Finding Terry

In March of 1998 I decided that I wanted to conduct a case study investigating how language is used in one elementary mathematics classroom. My continuing interest in language use across the curriculum fueled this decision. I was involved in research with two professors who were also exploring how language arts and mathematics fit together. I had spent time from November 1997 to June of 1998 in two elementary mathematics classrooms as a research assistant and was very interested in the diversity of practice in mathematics education.

Once I had decided on a question that I wanted to explore for my research, my first task was to find a classroom where I could 'live' for the following school year. I began to make inquiries as to who might be interested in such a collaborative study. Initially, I approached a former teaching colleague, Jane, with whom I had worked. We had both taught Grade 1 at the same school in the late 1980s and shared similar teaching philosophies and teaching styles. Jane was very busy and suggested another colleague I might ask. She was teaching kindergarten half time, had a young daughter at home, and really was not excited about the prospect of exploring communication in mathematics.

I then approached a Grade 3 teacher with whom I had worked collaboratively in the teaching of a university language arts course. Ida's elementary students had been paired with my university students on "writing projects," and Ida had an ongoing interest in collaborative work with the university. However, Ida, like Jane, was not interested in participating in the project because she was going to retire in two years and felt that a younger teacher would have more energy and would be more interested in the study.

By the end of April I realized that finding a teacher interested in this "collaborative endeavor" might be more difficult than I had anticipated. My two university advisors had both mentioned Terry's name because she had done work with them during her master's studies.

Terry stood out as someone who might be interested. I had met Terry the year before in a teacher education course at the university. There were 15 students in the class involved in reflective writing and the sharing of our teaching stories. Even though Terry and I had not been in the same dialogue group, the dynamics of the class created a feeling of trust among the students. Terry and I would often say a few words to each other at the beginning of class. We felt a common bond because we were both on study leaves from the same school board.

I phoned Terry in early May, briefly discussed my research plan, and asked if she might be interested. Terry was very positive about the idea but indicated that she would have to wait until she knew her teaching assignment for the following year before she could make a decision. I was relieved that she had not said "No," but at the same time I did not want to put any pressure on her. I asked her to phone me anytime in the next four months if she felt that she would like to discuss the possibility further.

In August I heard from Terry. I was in the process of getting ready for my candidacy, and I was concerned that I might not find an elementary classroom in which I could conduct my research. Terry and I arranged to meet at a coffee shop in Edmonton. It was two weeks before school was to start, and Terry was basking in those final days of summer before the onslaught of the beginning days of a new school year.

I took my proposal to the meeting and shared some of the basic premises of my study with Terry. She was interested in exploring how she used communication in her mathematics class. Our conversation took many turns that day. We discussed how we found ourselves situated in our teaching lives. Terry knew that she would have her hands full with a new Grade 6 class and felt that September would be devoted to getting to know her students. I left Terry with a copy of my proposal that day and asked her to read it and phone me with any questions she might have.

Terry phoned a week later and said that she was concerned that she would be put "on the spot." I realized I had hardly spoken of "collaboration" at all, assuming it was obvious. I had not been too clear in my communication. Terry's concern was with her "interpretation" taking priority in the study. I felt that it was important that we both go into this study collaboratively. I assured Terry that at no time did I want her to feel put "on the spot." Because of the initial trust that had grown between us, Terry said that she was confident that our research relationship would be collaborative. In our final interview at the conclusion of the study, I asked Terry about her initial reactions to the research. She responded:

> I think my initial worries were about the research. When I was first approached, it kind of made me feel incredibly responsible for teachers everywhere, that, you know teachers everywhere are going to be judged on what they saw happening, what you saw happening in my room. I thought, That's too much responsibility. So that kind of concerned me. Then when I realized and we talked about maybe doing some work together, you talked about perhaps, you know, we could see where the communication was and then talk about how we could improve it, I felt it would be a learning experience for me. I met you, and I knew that you were not a person to put pressure on me. I didn't feel any pressure at all. (Interview, May 5, 1999)

I knew very little about the process of communication in mathematics and had not taught mathematics using this new curriculum. I was a language arts educator and a reading

specialist. My MEd work had been in the area of writing in social studies, and during that time I had been working as a reading specialist for the school board. This doctoral research study would prove to be a journey for both the teacher and me.

Two months later I began the case study in Terry's classroom. Because I had to wait for the university's ethics approval, I was not able to begin the case study until November. This waiting period allowed Terry the time to get to know her students before I started my research. We had our first interview on Saturday November 14, 1998, and I started the classroom-based research on November 20, 1998.

Data Collection

The qualitative researcher can use a variety of research tools. I used interviews, classroom observations, field notes, and documents such as teacher resources and samples of the students' writing as my data-collection methods.

Interviews

Interviews played a significant role in my data collection. Becker (1986) reminds us that interviews can provide us with a rich source of data. She says:

> Researchers use interviewing because phenomenon they wish to articulate is deeply embedded in the relational and dynamic human world. . . . Because it is also dynamic and relational, the interview provides a human context that motivates the subject to take up the task of articulating complex lived experiences. (p. 102)

I conducted 10 open-ended interviews with Terry. These interviews were guided by questions and ideas that I wanted to explore and were based on my observations in the classroom and reflective notes I had written to myself. Patton (1990) describes this type of interviewing technique as the "general interview guide approach" (p. 280). The

interview guide serves as a basic checklist to make sure all relevant topics are covered. The wording and sequence of questions were determined in the context of the interview (Merriam, 1988; Patton, 1990). Bogdan and Biklen (1998) say that the researcher usually knows the subjects through interacting with them before interviewing, and the interview is like a conversation between friends. Terry and I would often talk about what Terry had done in mathematics class and move into the realm of conversation.

The first interview was conducted with Terry on November 14, 1998. This interview was of an exploratory nature; I asked Terry about her personal and teaching background, her teaching philosophy, her experiences teaching mathematics, her knowledge of curriculum, and her ideas about communication.

The remaining nine interviews were conducted in Terry's classroom at the end of the teaching day. We both agreed that it was important that we touch base every few weeks in the form of a taped interview because it was difficult to find time "just to talk" within the school day. These nine interviews spanned the time period December 14, 1998, to May 5, 1999. Questions in these interviews were guided by themes that emerged as a result of my fieldwork in the classroom. I asked many questions about why Terry had made specific decisions during mathematics class. The interviews varied in length from 24 to 60 minutes.

One of these interviews took the form of a planning session in which Terry and I discussed a mathematics unit on "data analysis." My purpose was to capture the collaboration and sense-making of two educators as they sought to interpret the mathematics curriculum. This proved to be the most elaborate planning session of the study.

The final interview was guided by questions of a summative nature. Terry and I discussed our time together and the sense we had developed about "communication" in mathematics by this point in the study.

All the interviews were taped and transcribed by a transcriber soon after they were conducted. Terry was provided with a copy of the transcripts, giving her the opportunity to read and clarify what she had shared in the interview. Terry stated that she enjoyed reading the transcripts and that it was fun to read over what we had talked about.

Participant Observation

Participant observation was a crucial means of data collection in my case study. To understand fully the complexities of many situations, direct participation in and observation of the phenomenon of interest may be the best research method (Patton, 1990). Spradley (1980) explains the role of participant observer as "the participant observer comes to a social situation with two purposes; one to engage in activities appropriate to the situation and two, to observe the activities, people, and physical aspects of the situation" (p. 54). I was engaged in both purposes in Terry's classroom: I engaged in activities and at the same time observed and took extensive field notes of the classroom setting. I visited and was a participant observer in Terry's mathematics classroom approximately two to three times a week from November 20, 1998, to May 4, 1999.

My role as participant observer was negotiated as time went on. Bogdan and Biklen (1998) caution that participant observers in classrooms have challenges as to how much involvement is necessary or expected by the teacher and children. They write:

> There may be pressure, some brought about during negotiating access, for the second adult in the classroom to function as a teacher's helper. As we already suggested, a certain amount of this type of participation can work, but you must be on your guard not to let it dominate your time. (p. 88)

I became very much a part of life in the classroom. For the first few visits I spent most of the time observing, but by the third visit I became a second adult educator in the classroom, talking about and helping the students with their work. I distinctly remember a day in late January when one of the students came over to me and asked me to help him with his data-analysis project; from that day on I felt truly accepted as a member of that classroom.

Field Notes

Field notes are central to participant observation. Bogdan and Biklen (1998) state, "The successful outcome of a participant observation study in particular, but other forms of research as well, relies on detailed, accurate, and extensive field notes" (p. 108). My classroom observations, discussions with the teacher and students, interview notes and transcripts, document reviews, and personal reflections were captured in my field notes.

Bogdan and Biklen (1998) describe two kinds of field notes: descriptive and reflective. Descriptive field notes provide a word-picture of the setting, people, actions, and conversations. Reflective notes capture the observer's frame of mind, ideas, and concerns. I used both types of field notes.

The descriptive notes were written in response to everything that I encountered in the research setting. As suggested by Bogdan and Biklen (1998), these descriptive notes encompassed portraits of Terry and the students, reconstructions of dialogue, descriptions of the physical setting, accounts of particular events, depictions of activities, and

descriptions of my behavior. I always tried to read back my field notes on the same day to get a sense of what themes I saw emerging.

I wrote reflective notes. These were personal in nature and provided me with the opportunity to reflect on my research experiences and record personal ideas, feelings, and problems. They were an extremely helpful research tool because they enabled me to work through some of my deliberations and allowed me the opportunity to take an introspective view of the research experience. Bogdan and Biklen (1998) caution us that in order to conduct a good study, the researcher must be self-reflective. They suggest that the researcher try to include reflections on analysis, method, ethical dilemmas, conflicts, one's own frame of mind, and points that need clarification in these reflective notes. As time went on, my reflective notes served as both a vehicle for exploring my feelings and a way for me to plan the course of the research. Many times I would write in my reflective journal after reading over my field notes; other times I would write an entry in the middle of the night when something struck me.

Documents

Qualitative research frequently uses documents as a source of data. Documents in qualitative inquiry yield excerpts, quotations, or entire passages from organizational, clinical, or program records; memoranda and correspondence; official publications; and reports (Patton, 1990). For this research I used the WCP document (Alberta Education, 1996); the currently authorized mathematics textbook series that Terry uses, *Quest 2000* (Wortzman et al., 1997); newspaper articles; handouts that Terry gave her students; and a note that Terry wrote to me at the end of the study.

I also used some student work samples that provided a springboard for conversations with the teacher regarding her practice. At one point we asked the students to write down an answer to the questions, "Write about mathematics, what you like about math, what you dislike. Is anything different from other years? When you have a problem, does it help when you discuss it with classmates?" Great care was taken to ensure that I had permission from the participants to use some of their written work as part of my data collection.

Data Analysis

The culminating activities of qualitative inquiry are analysis, interpretation, and presentation of findings (Patton, 1990). The qualitative researcher uses inductive analysis, which means that categories, themes, and patterns come from the data. Analysis will become a search for patterns, striving for workable categories. Stake (1995) says, "The search for meaning is a search for patterns, for consistency, for consistency within certain conditions, which we call 'correspondence'" (p. 78). I began my analysis during data collection, as recommended by Bogdan and Biklen (1992), Stake (1994), and Merriam (1998). Stake writes, "There is no particular moment when data analysis begins. Analysis is a matter of giving meaning to first impressions as well as final compilations" (p. 71). The analysis of data in fieldwork is not a distinctly separate phase. As Hammersley and Atkinson (1994) assert:

> It begins in the pre-field phase, in the formulation and clarification of research problems and continues into the process of writing up. Formally, it starts to take shape in analytic notes and memoranda; informally, it is embodied in the ethnographer's ideas, hunches, and emergent concepts. In this way the analysis of data feeds into the process of research design. (p. 174)

I always read back my field notes on the day on which they were written and made notes in the margins about ideas and hunches I had. After my interviews were transcribed, I would read back the interview and write initial speculations in the margins as I read.

After my data collection had ended, I began further data analysis by reading through my research proposal. Goetz and LeCompte (1984) suggest that one begin analysis by reviewing the research proposal. After this, I read through my field notes, interview transcripts, and documents, as recommended by Bogdan and Biklen (1998), Patton (1990), Tesch (1987), and Merriam (1988).

As a way to manage my data, I developed a coding system. Bogdan and Biklen (1992) write, "You search through your data for regularities and patterns as well as for topics your data cover, and then you write down words and phrases to represent these topics and patterns" (p. 166). Lincoln and Guba (1985) refer to this as "unitizing" the data. Examples of words and phrases were *physical layout, teacher talk, student talk, manipulatives, math vocabulary, assessment, guiding, extending thinking, teacher encouragement, collaboration, math curriculum, open-ended questions*, and *need for structure*.

I then searched for common and unique themes in my coded data. Researchers see the activity of devising common themes as "clustering" or gathering statements that are conceptually similar. I developed conceptual categories that interpreted the data. The conceptual categories I extrapolated were teacher planning; program of studies; curriculum resources; real-life teachable moments; collegiality; beliefs; the use of teacher talk for guiding, responding, and explaining; writing; student-response talk; communication in mathematics; my interpretations of language in mathematics;

assessment; pressure of tests and curriculum; and teaching philosophy. Using Merriam's (1998) idea of reading the first interview, the first set of field notes, and the first document collected, I went through the data in chronological order to come up with my categories. Patton (1990) reminds us that the naturalistic evaluator works back and forth between the data and the classification system to verify the meaningfulness and accuracy of the categories and the placement of data in categories; this I did throughout the analysis. I kept my research question in mind and asked myself if the conceptual categories emerging were plausible given the data from which they emerged. Janesick (1994) reminds us, "Staying close to the data is the most powerful means to telling the story, just as in dance the story is told through the body itself" (p. 215).

I took my analysis of the data back to Terry to share my perceptions with her. Stake (1994) advises that it is important for participants to receive drafts of how they are presented, quoted, or interpreted; and for the researcher to listen well for cries of concern. Terry did not suggest any changes and seemed pleased with what had been written. She shared, "I think the dissertation reads very well. . . . I discovered I don't speak in complete sentences. I guess I will have to self-monitor more closely."

The end product of my analysis, as defined by Merriam (1988), was a second-level analysis in which, rather than just describing, using my themes, I also interpreted what happened. Merriam refers to this as well as "category construction" and cites Glaser and Strauss (1967), who labelled this the "constant comparative method of data analysis" (p. 36). I linked the conceptual categories and themes by describing and commenting on what the data revealed in this study. Van Manen (1990) tells us, "If one uses themes to organize writing, then the challenge becomes how to treat each of the themes

systematically, even though one theme always implicates the meaning dimensions of other themes" (p. 168). Guba and Lincoln (1994) assert:

> The reader cannot be compelled to accept our analysis, or our arguments, on the basis of incontestable logic or indisputable evidence; we can only hope to be persuasive and to demonstrate the utility of our position for, say, the public policy arena. (p. 108)

A basic tenet of case study analysis is that the readers will also make their own generalizations given a good description of the research (Stake, 1994).

Triangulation

Patton (1990) writes, "One important way to strengthen a study design is through data triangulation, or the combination of methodologies in the study of the same phenomena or programs" (p. 187). I triangulated my data sources such as field notes, interview data, and documents. Patton points out:

> The term *triangulation* is taken from land surveying. Knowing a single landmark only locates you somewhere along a line a direction from the landmark, whereas with two landmarks you can take bearings in two directions, and locate yourself at the intersection (Fielding and Fielding, 1986: 23). (p. 187)

I found the same themes emerging from my field notes and interview data. Patton (1990) comments that triangulation of data sources within qualitative research will seldom lead to a single, totally consistent picture. He goes on to say, "Consistency in overall patterns of data from different sources and reasonable explanations for differences in data from divergent sources contribute significantly to the overall credibility of findings" (p. 467).

Plausibility

This study was limited to one teacher and her classroom, and I analyzed only the data from this context. Therefore, no broad generalizability can be claimed for the findings of the study. Guba (1981) says:

> The naturalistic paradigm rests on the assumption that generalizations are not possible, that at best what one can hope for are "working hypotheses" that relate to a particular context. . . . The issue here is not which assumptions are "true" but which offer the best fit to the phenomenon under study. (p. 77)

Connelly and Clandinin (1990) write about the notion of plausibility, stating, "A plausible account is one that rings true. It is an account of which one might say, 'I can see this happening'" (p. 8). In my study I hoped to offer one plausible way of looking at and interpreting a teacher's understanding and implementation of the process of communication in mathematics.

I wrote a clear and detailed description so that other researchers might use the study as a basis for comparison. Stake (1995) observes, "The methods for casework . . . are used to describe the case in sufficient descriptive narrative so that readers can vicariously experience these happenings, and draw their own conclusions" (p. 243). Given my research context and the data collected, I created a final document that accurately describes the research experience.

Limitations

This study was limited to one specific classroom. I looked at one teacher's interpretation of the communication process of the mathematics curriculum. There were limitations of time, how much time I was able to spend in Terry's classroom.

This study looked at one small part of a teacher's mathematical practice: how a teacher interpreted and put into practice the process of communication from the WCP (Alberta Education, 1996). I was not looking at other aspects of classroom pedagogy and therefore could not make statements or conclusions that address other areas of the curriculum.

Ethical Considerations

Concern regarding the anonymity of the participants and the confidentiality of the information they provided was given priority. All participants were asked to sign a consent form. Terry was informed that if questions asked in the interviews made her feel uncomfortable, she did not have to answer them. I explained to Terry and the students that if at any time they no longer wished to participate in the study, they were free to withdraw. All the tapes, transcriptions, computer disks, and my reflective journal were kept in a safe place in my home. Participants were given pseudonyms in the report, and every effort was made to lessen the risk of identifying participants in this study.

CHAPTER 4

CONTEXTUAL CONSIDERATIONS

The instructional decisions that Terry made were a result of many factors, and I borrow from several frameworks in the field of teacher knowledge and change to illustrate this point. Dewey (1931/1985) states, "The most pervasive fallacy of philosophic thinking goes back to the neglect of context" (p. 19). Fennema and Franke (1992) write about the many facets of teacher knowledge that influence what a teacher does and what students learn in the subject of mathematics. The relationships are dynamic and are developed in an interactive milieu. Variables include knowledge of mathematics, knowledge of mathematical representations, knowledge of students, and general knowledge of teaching and decision making. Jones (1997) speaks about the importance of context in studying the culture of the mathematics classroom. Several faces of context such as policies, school norms and expectations, perspectives on mathematics, beliefs about oneself as a teacher or mathematician, participation in teacher development projects, presence or absence of reform initiatives, existence and effectiveness of school councils, classroom resources, and collegial relationships can inform teacher change in the mathematics classroom. The professional knowledge landscape both inside and outside the classroom has a bearing on what a teachers does and knows (Clandinin & Connelly, 1995).

In this chapter I describe the many contextual factors that worked together to create Terry's professional knowledge landscape. I describe Terry's teaching history and memories of mathematics as an elementary student. I also discuss the students' perceptions of mathematics and the theoretical knowledge packaged for teachers in

textbooks as part of this professional knowledge landscape. Based on Fennema's and Franke's (1992) model, I describe Terry's knowledge of mathematics, her general knowledge of teaching and decision making, and her knowledge of students. Based on Jones's (1997) work on context, I describe Terry's participation in professional development and the classroom norms. I've broadened Jones' notion of school norms to include classroom norms (Yackel et al., 1991). I address the policies of the province and school board under Jones's umbrella. To conclude, I address the contextual feature of my presence as a researcher and the research project.

Terry's Teaching History

Terry started teaching as a young woman, having graduated from university at 19 years of age. She was eager to start teaching, and at the time of our research she had been teaching for 25 years. Her first teaching assignment was in a private kindergarten, and she was one of the first teachers to implement kindergarten into the regular school system. She taught Grades 1 and 2 for many years. At the time of the research Terry had spent the last seven years teaching at the Grades 5 and 6 level and had worked as a resource teacher and special-needs coordinator. She had been at her present school for five years.

Memories of Mathematics as an Elementary Student

Terry's earliest memories of mathematics as an elementary student involved standing in front of her Grade 1 class and counting from 1 to 100. She talked about her teacher pointing to the numbers on the 100 chart with a pointer. Another math memory was when the year changed from 1959 to 1960. She remembered being "abysmal" at probability in high school and not enjoying math at that point. Terry had a difficult time

with long division and has memories of her father helping her with the "short way." Terry talked about not having a very good idea of how number systems worked when she was in Grade 4 and recounted:

> I didn't really have a concept of number systems and numbers and how they are all connected and how they work. I think as you grow up and you have more real-life experiences with math, like, I mean, figuring out a bank book, a chequing account, simple addition, subtraction, doing something like measuring your house for wallpaper, or windows, or flooring, or you know, all those kinds of mathematical things, estimating distances in your car, and so now numbers make a great deal of sense to me. (Interview, Nov. 14, 1998)

Terry says that she now enjoys mathematics, and this happened once she started teaching Grades 5 and 6 and began to see patterns in numbers. She states, "I would love it if all I had to teach was math." I don't believe Terry had the time to put into planning mathematics lessons that she would have liked.

Terry seems to have a knowledge base and confidence with the Grades 6 mathematics concepts that I don't have, and I looked to her for guidance many times during the study.

The Students and Their Perceptions of Mathematics

Clandinin and Connelly (1995) write about the professional knowledge landscape that exists both inside and outside the classroom and of its being comprised of relationships among people, places, and things. The students were an integral part of the landscape of the classroom. There were 23 students in Terry's classroom, 16 boys and 7 girls. The students lived in an area that was considered a lower-middle-class neighbourhood. Terry considered her class a "united nations." (Fieldnotes, Nov. 24, 1998). Many of the students were living in single-parent and blended-family situations.

The deliberations and discussions that Terry and I had always focused on the students, the work they were doing, and our engagement with them. Students are increasingly playing a more active role in the mathematics classroom and contribute to the processes that combine to make up its culture. The most immediate and significant "other role group" for teachers are their students, who exert "the most important external constraint upon them" (Bishop & Nickson, 1983, p. 15).

Steffe (1991) points out that the teacher's plans must be informed by the mathematics of the students, stating, "The most basic responsibility of constructivist teachers is to learn the mathematical knowledge of their students and how to harmonize their teaching methods with the nature of that mathematical knowledge" (p. 17). An ongoing struggle for Terry is to ascertain where individual students are in their mathematical knowledge. As she said, "You can think you've gone through a concept and they won't ask for help, and all of a sudden you realize one person doesn't have a clue about anything that's been going on."

Near the end of the study Terry and I realized that we really had not asked the students about what they were thinking about mathematics. We realized that the harmonization of our teaching methods with the students' mathematical knowledge had not been easy or necessarily successful. We decided to ask the students for their interpretations of mathematics, to see if their experiences had been different from other years, and whether the discussion we tried to promote in class had been successful. We put the following statements on the board: "Write about mathematics, what you like about math, what you dislike. Is there anything you did differently from other years?"

Terry added, "When you have a problem, does it help when you discuss it with classmates?" The students gave us all types of responses. Kerri wrote:

> I think math is pretty fun. I like doing things like multiplication and addition because that is what I'm good at. Sometimes math is confusing for me. I don't like changing numbers into fractions, decimals, and percents. I like finding volume and area. This year we have done a lot more math than other years. (Documents, May 4, 1999)

Nadia wrote:

> Math is boring. I wish we never had to do that! I think discussing a problem with the whole class is a good idea to get everyone's opinion and discover new things and methods, you get the message around by discussing it openly. (Documents, May 4, 1999)

Alan wrote:

> I love adding and subtracting, geometry, and word problems. I work great with other people and hate digits and decimals. . . . I love making floor plans of homes but I don't really like finding area. I thought the cities we made with cubes, triangles, prisms, cylinders, rectangles, and pyramids were neat. (Documents, May 4, 1999)

Stephen wrote:

> In Math, what I like about math is that it's fun sometimes like when we did projects like when you ask a question on someone and then you make a line plot. . . . What I also like about math is that you can work with anyone you want to and you can talk and work and that way the work finishes faster. (Documents, May 4, 1999)

Brandon wrote:

> I like doing lots of things in math but overall my favorite thing is multiplying, I don't like division, I don't know why. It's easy but I just don't like it. I think it is a little bit easier to discuss math problems with classmates because then they might have a good idea and then we can do the same. (Documents, May 4, 1999)

Kevin wrote:

> I learned mixed numbers and I hate how its everyday. I can't solve a problem with a group, they do all the work and I barely learn. (Documents, May 4, 1999)

Jason wrote:

> To me math is fun because it is challenging, sometimes it is easy. My favorite topic in math is angles because they are easy. I think when I get stuck on a problem, talking to my neighbour about it doesn't help me because they could be really smart and you would get confused. (Documents, May 4, 1999)

Kim wrote:

> I like some of the activities we do sometimes like when we were doing area and perimeter designing a dream school that was fun, also percents and ratios, making office buildings etc. (Documents, May 4, 1999)

Michael wrote:

> Math can be hard if you let it be hard, but sometimes things can be frustrating and difficult to learn. (Documents, May 4, 1999)

The students all had different ideas about mathematics, what they found easy and what they found difficult. Students had varying ideas about talking problems out with their classmates; most found it helpful, whereas some found it too overwhelming. Desforges and Cockburn (1987) discovered that children in their study tended not to rate discussion sessions in mathematics very highly and suggest that perhaps children saw them as a distraction from the main agenda. Some of the students mentioned enjoying the projects that we did, such as the cityscapes and school mapping projects. The students were all bringing different histories into the mathematics classroom, and this added to the dynamics of our mathematics class and the way in which we individually and collectively came to make sense of the environment.

The children's attitudes toward mathematics impacted their work and just as importantly affected the decisions Terry made with respect to teaching. Some students did not enjoy talking about mathematics with their classmates and were uncomfortable when working in a group situation and talking about their math. Clandinin and Connelly (1988) write that a teacher's practical knowledge is influenced by what is experienced and lived within the classroom by the teacher and the students. Jones (1997) and Schifter and Goldsmith (1997) assert that integral to notion of context and teacher change are the students' understandings of mathematics. Observation of the students' mathematical thinking causes a teacher to reflect on her practice.

Mathematics Textbook

Clandinin and Connelly (1995) write about the professional knowledge landscape being comprised of the "theoretical knowledge" that is packaged for teachers in textbooks. Terry used *Quest 2000* (Wortzman et al., 1997), which is one of the recommended resources for implementation of the new mathematics program of studies in Alberta. She used it almost exclusively, saying at one point:

> There are certainly things I like about this. I like, there are many ideas, but you get so many different books. You have a teacher's guide, you have the solutions manual, you have a technology packet, you have the problem of the week book. Then you have another extra workbook because a lot of teachers felt that there wasn't enough drill and practice. . . . So we do use *Math Quest 2000* a great deal. I like the program. (Interview, May 5, 1999)

(The extra workbook was put together by a group of teachers from the school district who believed that the students needed more drill and practice.)

Terry used the teacher's guide to help her plan her lessons. The guide has a list of program principles that provide a philosophical rationale for the series (see Appendix C).

The term *constructivism* is never mentioned in the guide but is inherent in Program Principle 1, which states, "Mathematics is a natural activity to children, and children learn mathematics best when they construct their own understandings" (Wortzman, 1997, p. x). Program Principle 7 states, "Communication is core to all mathematics activity. Communication is promoted through groups working cooperatively and is enhanced by children writing about their mathematical ideas, processes, and questions" (p. x). Group work is emphasized in the teacher's guide as a major part of every lesson.

There is a strong emphasis on writing in the teacher's guide. Wortzman et al. (1997) state, "The authors of *Quest 2000* view written communication as a powerful learning strategy. Throughout the program the students are given many opportunities to write about their thinking and learning" (p. xxi). Two pages are dedicated to this in the guide. An excerpt says, "Writing helps students reflect on their experiences, their thinking, and their understandings. It enables them to reframe newly-acquired knowledge in their own words or in pictures, and thereby to deepen their understanding" (p. xx). Whole-class and individual-student journal writing are described and emphasized frequently throughout the student textbook. Talking and reading are not addressed as individual learning strategies in the teacher's guide. There is a strong emphasis on the teacher's delivery of mathematics vocabulary. For example, the authors state, "Ask students: How might a sample not accurately represent the entire group? Explain that the entire group is called the *population*, and that when the sample is not representative of the population it is called a *biased sample*" (p. 224). The textbook provides questions to ask the students, and Terry carefully adheres to such questions.

Even though there is a strong emphasis on writing in mathematics in the textbook, Terry did not utilize this aspect of the program. The emphasis on teacher provision of mathematical terminology is part of Terry's instructional repertoire, and the questions in the textbook promote Terry's teaching style.

Terry's Beliefs About Mathematics Education

Terry's beliefs about mathematics were related to her general teaching philosophy. Educators now realize that how teachers interpret and implement curricula is influenced significantly by their knowledge and beliefs (Cork & Peterson, 1986; Romberg & Carpenter, 1986; Thompson, 1992). When I asked Terry about her beliefs about mathematics, she talked about such issues as problem solving, calculators, the need for quiet time, the need to encourage excitement in the students, and the importance of hands-on manipulatives. Terry often links her beliefs about mathematics to the notion of problem solving and hands-on work. According to Kuhs and Ball (1986), a constructivist view of mathematics learning is the instructional model most likely to be advocated by those who have a problem-solving view of mathematics. When I asked Terry about her beliefs about mathematics, she responded:

> Well, you know, sometimes I really believe math has to be done through problem-solving situations, but sometimes you need 15 minutes of peace and quiet and you pull out a sheet of subtraction, division, or multiplication and say, "Here." You know the kids like to do that stuff every once in a while. . . . It is good practice, so sometimes you can rely on math to give you that 15 minutes of down time. (Interview, Nov. 14, 1998)

In our last interview on May 5th, Terry again touched on the notion of problem solving and manipulatives. She questioned how much depth of understanding some of the students really had with respect to solving problems and added:

> So theoretically I would love to have kids working with manipulatives all the time and exploring math that way all the time. In a very ideal classroom with an ideal group of kids we could do that, because that's the way kids will learn about math. So I believe that math goes from the concrete to the pictorial to the abstract. I don't always teach that way, and I'll be honest about that, but that's my belief. (Interview, May 5, 1999)

Terry believes that students should be allowed to use calculators whenever they want to and said that at the beginning of the year she would take time to review how to multiply and divide without calculators because she wanted to make sure that they knew their basic facts.

Terry believes that in mathematics we have moved students too quickly from the pictorial to the numerical stage. She related:

> Teachers feel the pressure to get busy with numbers and adding and all when the kids may have not learned yet that one and one makes the story. If they understood the pictorial very well, then I think we would look at it differently and the kids would look at it differently too. (Interview, Jan. 21, 1999)

Terry talked about the frustration she experienced with students being at varying levels in their understanding of mathematical concepts:

> I'm struggling with how best to make sure that everybody has the concept. I'm finding with this group too, I have some very high and then I have some other ones, and you can think you've gone through a concept, and they won't ask you for help and all of a sudden you realize one person doesn't have a clue about anything that's been going on. (Interview, Jan. 21, 1999)

Terry seems to have a good knowledge base and realistic attitude towards mathematics. She understands the importance of problem solving to mathematics education and is aware of her limitations with respect to this. Some of her students' difficulties with mathematics are cause for concern, and this affects how she teaches.

Knowledge of the Process of Communication In Mathematics

Terry's beliefs about the importance of talk are not reflected in her everyday classroom practice. Talk is initially the essence of what the process of communication means to Terry in her Grade 6 mathematics class. Terry explained this in our very first interview. I asked her what the process of communication meant to her. She answered:

> The first thing I think of is speaking, and maybe the flip side of that is listening, and I think for mathematics, definitely, it's talking about what they're doing, making sense of what they're doing, explaining to somebody else. (Interview, Nov. 14, 1998)

Terry has difficulty putting her beliefs about communication into practice. Later on in the study she acknowledged that her beliefs and practice are not always aligned.

Terry's Teaching Philosophy

Terry believes that every student is unique and creates his or her own knowing: "I believe that every child will take something different away from a lesson because they will construct their own knowledge based on their own experiences." She believes that most children really want to learn and that her role is to facilitate and help the children become enamored with and passionate about learning. She told me, "When I read a story I want them to feel passionate about a story; or when they do a math problem, I want them to have stick-to-itness, the creativity to look at it in a different way, not just to say, "I can't do it' and give it up."

Terry considers herself a constructivist teacher. According to Terry, constructivism means giving the students lots of opportunities for discussion. She had taken a course during her master's work on constructivist teaching and learning. She always feels the tension between what she thinks of as traditional modes of instruction

and student-oriented approaches to teaching, saying, "I like the quiet, structured day when I know what's going to happen, when everybody's working quietly, and yet I like the discussion and the ideas, and I like, you know, I'm fighting with that, for myself, a great deal." She went on to say:

> There are elements of me that are still very traditional, you know. I like it quiet when we work. In some ways I don't have the time to allow the exploration because I think really in constructivism you would have simply said, "How are you going to go about this? Think about what you know about percents," and you would have let the kids explore, and go through trial and error. I'm finding that I have to move slowly in that direction too because these kids are really used to instant gratification. And if something doesn't work right away, well, what's the point? Forget it. So I guess for me, moving away from me, standing in front of the room having all the answers to more discussion, more exploration, taking their ideas, and maybe like even one boy said to me one time, "How come we always have the same agenda?" I started thinking about that and thought, Yeah I don't like the way my day is even laid out. . . . And so what I have started doing is doing LA and social studies in the morning and then math and science in the afternoon. (Interview, Nov. 14, 1998)

Terry talked in that same interview about the importance of discussion in a constructivist classroom. She sees this as integral to promoting students' meaning making. In these initial discussions I was sensing the conflict for Terry between traditional practice and what she considers to be constructivist practice.

In the final interview of the study on May 5th, Terry talked at great length about her teaching philosophy. The following is Terry's response to the question of what her teaching philosophy is:

> I've thought about this, and I don't know whether my teaching philosophy has changed or my teaching style, but I think my philosophy is tied in with beliefs about what children need and how they learn. So I believe children need structure with enforced boundaries, and then within the boundaries they have the freedom to explore and make decisions and learn. I think they have to be aware of your expectations; that's behavioral, and so that also ties in with the boundaries and the academics. I think that self-esteem for the students will follow when they're successful at their tasks. And of course, I think that school has to be a safe

> environment for anyone, students and adults. I think it has to be the same not only physically but also emotionally. Tied in with this is the willingness to take risks, which if you're never willing to take a risk, I don't think you learn. (Interview, May 5, 1999)

Terry is aware that her teaching philosophy and actual practice are not always aligned. A few years earlier we had taken a course on the topic of teacher reflection together, and writing about our teaching stories was an important part of the course. Terry referred to this in one of our conversations:

> I think some days my teaching style does not always reflect my philosophy, and what has happened for me in the last couple of years is, I've really had to start looking at my teaching stories and through looking at these stories examine what my beliefs were; . . . and oftentimes the stories were not reflective of what I said my beliefs were. That causes great angst to a teacher, I think, when those two are not connected. So what I've been trying more and more to do is put my philosophy and my style, make them so they're as similar as possible. So I said I think my teaching style is old fashioned on one hand, but I think it's sort of eclectic on the other. I'm not a bandwagon person, have never been. I really try to pull the good I see from ideas that have come up, which is the newest and latest and hottest thing, so I will try; but I also am not afraid to stay with the tested and true. So if my kids work better in rows, if they're able to concentrate better in rows, then put them in rows. So I don't have any problem with kind of, you know, physical arrangements. I mean, my physical arrangement right now is very different, but yet it works. So I guess that's what I try to do in my teaching style. I try to respect the children as learners, as people. I expect them to respect me, so I have to respect them. (Interview, May 5, 1999)

Terry's classroom practice seems to be a reflection of her teaching philosophy: on one hand "old fashioned," as she referred to it, and on the other eclectic. She has given her teaching philosophy a great deal of thought and isn't afraid of the contradictions between her philosophy and practice.

Terry's Beliefs About Students

Terry has the utmost respect for her students, and that is demonstrated on a daily basis. It is very important for Terry that her students go on to Grade 7 capable of solving their own dilemmas. Terry continually supports her students' self-esteem and confidence. She shared:

> It's one of the most essential things that children can learn, is how to solve their problems, because they're not going to have an adult there all the time to help them. If they can solve some real-life issues here, that gives the experience. It gives them, hopefully, success, and they can build on that, and they'll feel good about it. And I think success will also breed self-confidence and self-esteem. You know, sometimes you have these silly self-esteem lessons that are kind of unrelated to anything you're doing, whereas kids need to try something. They need to fail sometimes, know how to deal with that. They need to succeed sometimes, and if they succeed, then they feel comfortable, and that builds self-esteem. Achievement builds self-esteem, I believe. (Interview, Dec. 4, 1998)

Terry respects her students as constructors of their own knowledge. In one interview she commented:

> These kids have so much knowledge that we never tap into. We're always sort of telling them, "This knowledge," instead of saying, "What do you already know?" and then go on from that basis. It sounds so simplistic; we should be doing this all the time. But I think a lot of times it tends to be in school, not my philosophy, but the way I was taught was all top down: "Here is the expert knowledge that you need to know in order to tell it back to me so that I can give you a mark." I thought, These kids really have a lot of good knowledge, and if we start with that, we could say, "And then what about this and this and this?" So it's an add-on and it's a connection as opposed to being something so removed from their own worlds. (Interview, Dec. 16, 1999)

Terry sometimes talked about her desire to understand how her students were conceptualizing mathematics and how they learned in general. She respects the fact that many times they think beyond the boundaries of what she thought is possible.

It is important for Terry that she help the students to relate to real-life situations to help with their understandings in math and other subject areas. I equated this to a type of

respect Terry had for her students, a respect for their background experiences. I asked her about her tendency to relate her teaching to real-life events. She answered:

> There are so many moments during the day that if you kind of know what you want the kids to learn, eventually you can capture those teachable moments. It doesn't have to be planned, and that comes with experience, and then tying it in with real-life, like, 'What's the purpose?" I've always looked for the teachable moment in whatever. I think last year was kind of eye opening for me, because I introduced a concept and it was in math and I said, "Now where would we use this? Where would we find it?" and the kids came up with a huge list. (Interview, Dec. 16, 1998)

There were numerous times that I noticed Terry relating concepts to real-life events during her teaching. It is an important part of her teaching style.

Terry's respect for her students and concern that they leave Grade 6 armed with the concepts they need for Grade 7 have a bearing on her instructional decisions.

Terry's Professional Development

Terry's ongoing interest in research and reflective practice was a strong impetus for her involvement in this study. She had taken only two curriculum and instruction courses in mathematics education while taking her undergraduate degree program. In preparation for teaching the new mathematics curriculum, Terry had taken a series of six Saturday morning inservices from a mathematics instructor at the University of Alberta. Terry described the inservices as "wonderful, because they really changed your mindset about how to even look at the new program of studies, I found that very, very helpful."

She enjoys philosophical meanderings and was involved in two other studies that year, one in science and one about teachers' beliefs about change. Terry talked about her reasons for involvement in research projects, saying, "Some people can think up there,

but then can't translate that. I want to be able to try, then translate that into practical knowledge and practice, but I think for me it's selfish."

Terry had just completed a master's degree in curriculum two years before. She believes that the experience broadened her definition of curriculum: "When I talk about curriculum, I talk about everything that happens to the student during the day, not necessarily the program of studies."

Terry's master's thesis explores students' attitudes towards report cards, and through her study she gave great thought to the meanings of *assessment* and *evaluation*. Terry's ongoing interest in research has stayed with her:

> When I came out of university I had such a positive experience. I think I had three people who were willing to work with me for my master's thesis, and you really need teachers who are willing to do that. The main reason is because research interests me, learning interests me, so I always want to keep my fingers in that pie. And I find I need to be involved with somebody who I can exchange ideas with. (Interview, May 5, 1999)

Terry also enjoyed being involved in the research because she enjoyed having the company of another adult in the classroom. On one occasion she stated:

> You know, sometimes teaching can be very lonely. . . . You're stuck in the room with all these kids, and sometimes you just need to roll your eyes or something to another adult. It's like another pair of eyes, because I can't see everything. (Interview, Nov. 14, 1998)

Terry is not afraid of honest reflection of her practice or of having a researcher present in her classroom.

Classroom Management

Terry's classroom-management style reflects her teaching style. She likes order and wants an environment where everyone is safe and aware of their boundaries. When we talked about the possibility of doing research together in August of 1998, Terry knew that her incoming students were going to be a challenging group, and she was conscious of her management style. I asked Terry near the end of the study about her classroom-management style. She explained:

> I think the very basis of my classroom management is trying to know each student and building a relationship with them so that if I've got a relationship with them, then oftentimes all it will take is a very low-key kind of intervention, a hand on the shoulder, a raised eyebrow, walking over to that person. Now that does not always work, but then if there are consequences that you—the kids know what they're going to be, that you enforce as consistently as possible and keep them as naturally as possible, I mean, we have our bad days when that doesn't work and you just say, "Get out of my hair. Go away. I need five minutes away from you." But for the most part it doesn't take that. You know, I think if you do build that rapport and relationship with them, they will usually respond. (Interview, May 5, 1999)

When I entered Terry's classroom I noticed how quiet and well behaved her students were. Terry uses a low-key approach to classroom management, and any conflicts are dealt with promptly. Terry uses a banking system with the students in which they keep bankbooks, and she allots them certain amounts of money based on points they have earned. Money is also taken away if they misbehave. They can then redeem their money for opportunities such as staying in and playing a game at recess. Terry's fairness and consistency with classroom management is noticeable on a daily basis and is very much an accepted norm.

Format of Mathematics Classes

Terry's mathematics classes followed a regular format. Mathematics always came after silent reading in the afternoon when the students had calmed down after their lunch break. Terry would then hold a review of the previous day's work. As Terry said, "It might be marking work that's been done or just standing up and talking, trying to get them involved." After that there was a mini-lesson in which a new concept or skill was taught, and usually this was tied in with a review of previous learning important to the acquisition of the new concept. Terry would usually involve the students in a teacher-directed discussion of the new concept, and the students would then work independently, answering questions from the textbook. Terry always tries to involve the students by giving them some sort of immediate feedback. She said at one point, "I try to give them fairly immediate feedback on how well they're doing. Like, if they finish, say, work in a book, then I like them to mark their own and see how they've done." The format of Terry's mathematics classes stayed fairly consistent throughout the study.

Context of the School

Midlands School was a 15-year-old public school in a middle-class neighbourhood, with approximately 360 students. Many of the students come from single-parent families, and there are many ethnic groups represented in the school. There were two classes per grade, except in Grade 5, where there was a Grade 5/6 combined class. There were three learning-disability classes.

The school atmosphere was warm and friendly. Upon entering the school, one was greeted by a caring administrator and office staff. There were lots of children's books and teddy bears in the office, and I was always a welcome visitor to the school.

Terry's classroom was in a portable attached to the school. This seemed to work well for the purposes of the Grade 6 class because there was an exit to the playground from the room, which seemed to give the students a sense of independence and reinforced their senior status in the school. There never seemed to be quite enough room in the classroom, and in my first visit the 23 students were seated in four rows. As the study progressed, the students were seated in many different configurations. There were times when the students were seated in pairs or groups of four or with the desks arranged in a semicircle. The room was cheery and bright. There was a row of windows along one side of the room, and science posters and world maps were on the walls. Below the windows was a series of shelves that housed the students' books and classroom materials. The students felt safe in this environment and seemed to be very much at home,

Policies and Reform Initiatives

Political Climate

During the two years prior to my study, Alberta had been experiencing a series of cuts to both education and health programs. Teacher morale had been low because schools were faced with the prospect of cutting teaching positions, and there was little money available for professional development opportunities and extra support within the schools. It was difficult to get release time to attend district seminars on mathematics because budgets were tight. During the winter when I was conducting my study, a headline in the *Edmonton Journal* (Unland, 1999) read, "$2.2M Aimed at Teaching New Math, Mar Reveals." The provincial government had announced that it would spend $2.2 million teaching teachers how to teach the new mathematics curriculum. An excerpt from the article reads:

> The money is meant for programs in the 1999/2000 fiscal year to help teachers learn new ways to teach mathematics and to assess student achievement. Principals, guidance counsellors and parents will also be eligible for training to help them support students. (p. 3)

The insertion of this money came about as a response to previous cuts in educational spending in the province and because of an interest in improving students' mathematics scores.

School District Policies

The school district priorities were also part of the contextual considerations. One of the district priorities for the 1999-2002 period was to "improve student achievement in the core subjects with an emphasis on language arts and mathematics" (Edmonton Public School Board, 1999, p. 3). From the standpoint of the school district, mathematics was a priority.

In the summer of 1998 the school district sent out a district mathematics survey asking teachers to rate on a response scale statements about the implementation of the new mathematics program of studies. Two of these statements were, "As a classroom teacher, I feel I understand the specific outcomes of the new math Program of Studies at the grade level I teach" and "I have sufficient strategies and resources to develop mental math skills in my students."

Terry was aware of the emphasis put on mathematics education by the school district, but we did not often talk about it. Teachers were reminded of the school board priorities on an ongoing basis. These were used for school budgetary planning and for schoolwide goals.

Provincial Tests

In the province of Alberta all students have to write the provincial achievement tests at the end of the year in Grades 3, 6, and 9. This can put a great deal of pressure on the teachers in these grades, and Terry is no exception. Test results are published in the newspaper for each school, and parents often choose schools and teachers based on this information. We had many conversations about the stress that the tests caused for Terry. We talked about the pressure to get through curriculum and about not having enough time to stay on a topic. Terry said:

> We speed through very quickly and there probably isn't the time for them to even construct meaning. . . . You know, we tend to always come back to the Alberta government and the Program of Studies and stuff like that, and I think that definitely as a teacher I do feel a great deal of pressure in Grade 6 because of the exams. . . . You get into division two, and especially there is pressure from tests, and I know people don't want to hear that. (Interview, Feb. 24, 1999)

In our final interview Terry again talked about not being able to give some of the subjects the attention they deserved. She revealed:

> I would love to have a year to integrate the curricular demands. . . . I would love to have time to focus on a particular subject area, and yet every day I've got to have something that will keep the kids motivated and on track and cover the curriculum in seven different subject areas. I find that I'm very fragmented, and so of course I cannot physically do the kind of job I want to do in every subject area. (Interview, May 5, 1999)

A concern for Terry is that the tests do not necessarily test the content of the curriculum. The questions are all multiple choice, and there is no opportunity for the students to be reflective. At one point in the year Terry had sat down and tried writing an old achievement test. She said:

> You know, I sat down and actually wrote them with the kids, and they are very difficult. . . . There are some test items I could have argued with slightly, and I thought these kids do very well to write these tests. Sometimes if you have more knowledge it's a detriment because you're reading beyond the question, and so now I remember some of those things, and now when I'm coming to them I go, "Guys, you know, I know that a question like that was on the test." (Interview, May 5, 1999)

I asked Terry what the driving force is for her as she plans her mathematics program. In her response one could detect her philosophy about teaching as well as the curriculum pressure she felt:

> I think the bottom line, the driving force, would be that I know I have to get through this all, and I want the kids to have a basic understanding of what's expected. And with that, I would like it to be as meaningful as possible for their lives so that when you introduce something you try and tie it in perhaps with something that's actually happening in their real life. (Interview, May 5, 1999)

Terry was very honest about the pressures that came to bear because of the provincial tests.

WCP

Terry has very definite ideas about what constitutes "curriculum" because she had done graduate work in that area a few years before our research project. To Terry, *curriculum* refers to everything that happens to children during the day, not just following the formal documents such as the WCP (Alberta Education, 1996).

However, we did talk about the WCP many times. The original question for my dissertation was, "How does one teacher interpret and put into practice the process of communication as identified in the WCP document?"

There were many times throughout my research that I asked Terry about her familiarity with the WCP and how and where she utilizes it. She told me that she usually

refers to it about six times a year. She always looks at it at the beginning of every year, at the beginning of every term, and at report card time as a check for the wording of report card comments. She said:

> Always at the beginning of the year, of course, then I kind of lay it out. Then I go back about report card time, sometimes for wording for report cards, sometimes just to make sure I'm on track. Then I always—sort of every term I revisit, and it's a way of keeping on track of a timeline. I think I do it for other reasons, as I mentioned, terminology, the explicitness of the terminology, and so wording for report cards, because what you want to do is, you want to report on curriculum, expectations. I like to go back to it for that. (Interview, Jan. 21, 1999)

Terry finds the curriculum strands and associated questions helpful. She acknowledged that it was very easy for her to lose sight of the WCP: "It's dangerous in a way as well, because sometimes you get hooked into the textbook or you get hooked into something else, and you really forget what the program of studies says they should know or shouldn't" (Interview, Jan. 21, 1999). This did not surprise me at all, because I remember as a teacher sometimes referring to the curriculum documents only once or twice a year. It depended on the number of times I had taught a certain subject and whether a document was new.

Constructivist perspectives contributed to the shaping of mathematics reform. The WCP is based on a constructivist perspective. The term *constructivism* is never mentioned in the document but is reflected in the statement:

> Students are curious, active learners who have individual interests, abilities, and needs. They come to the classrooms with different knowledge, life experiences and backgrounds that generate a range of attitudes about mathematics and life. Students must learn by attaching meaning to what they do; and they must be able to construct their own meaning of mathematics. (WCP, p. 2)

There are inconsistencies in the document with respect to the inclusion of the process of communication. Within the specific outcomes for the Data-Analysis Strand

and the Shape and Measurement Strand, there are opportunities for the communication process to be included, but it is not listed as an outcome in these strands. For example, one of the specific outcomes from the Shape and Measurement Strand is, "Develop, verify, and use rules and expressions for the perimeter of polygons." The mathematical processes that complement this are listed as the Connections Strand, the Problem-Solving Strand, and the Reasoning Strand. The Communication Strand is not mentioned.

Researcher and Research Project As Context

My presence in the classroom was an important contextual variable. According to Jones (1997), if we understand context to be vitally important to the meaning of a teacher's actions, then we must be concerned with our own poking about at contextual factors. A key part of research based on a constructivist paradigm is the importance of the researcher in relation to those she or he is researching. Guba and Lincoln (1998) reinforce the idea that individual constructions can be elicited and refined only through interaction between and among investigator and the respondents. Michelle Fine (1998) has written about the "other" in research and using the metaphor of "hyphen" emphasizes the importance of the equality of relationship between the researcher and those we research. Fine states:

> I mean to suggest that researchers probe how we are in relation with the contexts we study and with our informants, understanding that we are all multiple in those relations. I mean to invite researchers to see how these " relations between" get us "better" data, limit what we feel free to say, expand our minds and constrict our mouths, engage us in intimacy and seduce us into complicity, make us quick to interpret and hesitant to write. Working with the hyphen means creating occasions for researchers and informants to discuss what is, and is not, "happening between" within the negotiated relations of whose story is being told, why, to whom, and with what interpretation, and whose story is being shadowed, why, for whom, and with what consequence. (p. 135)

I was always wondering about how my presence was affecting the others in the classroom and therefore my findings. I always made every effort to ensure that there was open communication between Terry and me.

I brought my own beliefs and insecurities about mathematics to the project. In an interview with Terry I said:

> I'm still entrenched in the process of trying to come to my own beliefs about mathematics. Mine are so entrenched in that it's difficult and it's just to do with pencil and paper activities and I didn't take anything to do with mathematics at university. I stopped math in grade 10 and I guess part of the nice thing about doing, just touching research in this area is really dealing with my own ghosts and demons. I guess I'm trying to change my beliefs about math. I know I can't try to change but as I say mine are starting to change. (Interview, Nov. 14, 1998)

I will never know the extent to which Terry might have been responding to my worries.

My presence in Terry's mathematics class caused Terry to focus on the process of communication. Terry had agreed to be a participant in the study, but had I not been researching in her classroom, her instructional focus could have been much different. The knowledge that I would be in her classroom two to three times a week to try to answer my research question no doubt put pressure on Terry. The understandings and questions with respect to the process of communication were made possible by the many discussions we had during and after mathematics class. If we had not held these discussions, different aspects of Terry's practice might have been emphasized.

My presence no doubt had an impact on the students. I was another adult coming into their mathematics class a few times a week. The students' responses to Terry's questions, the degree of participation in their learning, and the quality of their work might have been influenced by my presence. Bodgan and Biklen (1992) comment on the dilemmas a researcher faces in the classroom in relation to the students: Does the

researcher act as an authority figure or as a quasi-friend? I believe the students looked to me as an authority figure, but I was always trying to transcend this and act as a co-constructor with the students.

One never knows to what degree he or she influences a situation, but if we truly believe in constructivist philosophy, we know that all members of a community play an integral role in constructing reality. In Bruner's words (1986), "I believe we construct or constitute the world" (p.130). There is no doubt that my presence affected Terry, the students, and what was learned. Jones (1997) mentions that questioning teachers about their practice creates a different context spawning different interactions and possibly different experiences of their teaching.

Summary

A consideration of the many faces of context serves to provide a foundation for examining the many complexities involved in Terry's instructional decisions. Terry's teaching history, memories of mathematics, the students' perceptions of mathematics, the textbook, Terry's beliefs about mathematics education, her knowledge of the process of communication, her teaching philosophy, her beliefs about students, Terry's professional development experiences, her classroom management style and the format of the mathematics class, the policies and reform initiatives such as the WCP (Alberta Education, 1996) and the provincial tests, and my presence as a researcher in Terry's classroom create a backdrop for coming to understand the decisions we made with respect to the process of communication in mathematics. The relationships among these contexts were dynamic.

CHAPTER 5

FINDINGS FROM THE UNIT ON ANALYZING DATA AND GRAPHS

Introduction

This chapter profiles the talk and writing that occurred in Terry's classroom during the study of the Statistics and Probability Strand that comprised Unit 6, Analyzing Data and Graphs, from the *Quest 2000* (Wortzman et al., 1997) textbook (see Table 1). I describe the focus on teacher-directed talk and emphasis on mathematics vocabulary instruction as reflected in six of Terry's lessons. I then discuss the data-analysis think-book writing project that created the backdrop for the most extensive student writing project of this research study. The findings are chronologically presented.

Table 1

Units Taught From *Quest 2000*

Dates	Units
Nov. 24- Dec. 16	Initial classroom visits
Jan. 5, 7, 12, 14, 19, 26, 27	Analyzing Data and Graphs (unit 6)
Feb. 9, Feb. 18, Feb. 19	Measuring and Analyzing Angles (unit 8)
Feb. 23, Mar. 9, Mar. 11, Mar. 18, Mar. 23, Mar. 25, Apr. 6	Exploring Geometric Solids (unit 9) Measuring Area and Perimeter (unit 11)

The following chart presents the general and specific learning outcomes for this unit addressed in Terry's classroom as found in the WCP (Alberta Education, 1996). These outcomes are listed in the WCP and are addressed in the Quest Unit. The symbols representing the mathematical processes that lend themselves to the teaching of the Specific Outcomes follow each outcome. For example, S.O. #1 lends itself to

communication (C) and reasoning (R). All five specific outcomes emphasize the process of communication.

Mathematical Processes

C- Communications
CN- Connections
E- Estimation and Mental Mathematics
PS- Problem Solving
R- Reasoning
T- Technology
V- Visualization

Strand : Statistics and Probability (Data Analysis)

Students will:

- Collect, display and analyze data to make predictions about a population.

General Outcome

Develop and implement a plan for the collection, display, and analysis of data gathered from appropriate samples.

Specific Outcomes

[#1] Formulate questions for possible investigation given a context. [C, R]

[#3] Select and use appropriate methods of collecting data:

- designing and using structured questionnaires
- experiments
- observations
- electronic networks

[C, PS, T]

[#4] Select and defend the choice of an appropriate sample or population to be used to answer a question. [C, R]

[#6] Display data by hand or by computer in a variety of ways, including:

- histograms
- double bar graphs
- stem and leaf plots [C, T, V]

[#7] Read and interpret graphs that are provided [C, E, PS, R]

In this unit the students were learning how to formulate questions, conduct a survey in order to collect data, analyze data, and use appropriate graphs to show their findings. They learned how to plot data on scatterplots, line plots, double-bar graphs, and stem and leaf plots, and how to interpret these.

Teacher-Directed Talk and Emphasis on Mathematics Vocabulary in the Data-Analysis Unit

The following are examples of teacher-directed talk that took place during the data-analysis unit. The format of Terry's math class was always the same. She began with a review of what they had done on the previous day, a review of important mathematics terminology, and the marking of homework. She followed this with a mini-lesson in which a new concept or skill was taught. Finally, there was practice usually focused on answering questions from the *Quest 2000* textbook (Wortzman et al., 1997).

The following excerpts are taken chronologically from my field notes for the month of January. As part of the data-analysis unit, we involved the students in a data-analysis project in which they were to pose a question and choose a population to survey. Part of this required that they write about what they were doing in a think book. The samples of talk from January 26th and 27th lessons took place after the introduction of writing for the think books.

January 5th Lesson

(All mathematics vocabulary from the textbook will be highlighted in italics. *T* refers to Terry, *S(1)* refers to the first student talking in the lesson, *S(2)* refers to the second student talking in the lesson, *S(3)* refers to the third student talking, *S(4)* refers to the fourth student talking, and *S(5)* refers to the fifth student talking.)

On January 5th, the second day of a new term, I started my visits to Terry's classroom again. As I entered the classroom, the following teacher-directed talk was taking place:

T: Take out your math books. [Each student took out a *Quest 2000* textbook.]

Terry then drew a horizontal line on the board and wrote the numbers from 0 to 6 below the line.

> T: Thanks to those sitting. I need your eyes up here. Yesterday I asked everyone to make a chart for their data about how you spend your time. We rounded it off to five hours. How many kept track about the time spent eating or shoveling? We had a category for TV, computers, and video games. I'm going to ask each one of you about this. I will organize this on a *line plot*. I will go up and down the rows asking you questions.

Terry then drew a line plot on the board. She asked the students about hours spent playing:

> S (1): I played for two hours.
>
> T: Let's round to the nearest half hour. If I go from 1:30, then another half hour is 2:00.
>
> S (2): Two hours.
>
> S (3): One hour.
>
> S (4): Two hours.
>
> S (5): Zero.

As the students answered, Terry put Xs on the line plot.

> T: We'll be keeping information for the rest of the week. By counting X's we know the number of people that responded. What conclusions can we draw from this simple line plot?

Terry then asked students about the number of hours spent watching TV.

> S (1): The *majority* watch one to two hours.
>
> T: Can you give me a *numerical* value?
>
> S (1): One, more than half.
>
> T: What is eleven, 50% of what? Any other *generalizations*, wrong word, what other *conclusions*?

S (2): Too much TV.

T: We get into value judgements. Could I say that the majority of all Grade 6 students watch less than two hours of TV a night?

S (3): No, because if someone is in another place, they might overrule this.

T: Are you saying this may be a *biased sample*? Remember, a *biased sample* is a sample of people that's not *random*. We could ask people from different countries. What he is saying is that this is a *representative* sample. What about the four people sitting near you? When would it be *biased*, when would it be *random*?

Terry then drew the students' attention to the questions and pictures in the textbook. There were pictures of different types of graphs such as pie graphs and broken line graphs. She pointed out the textbook pages that they would have to complete, and she read through some of the questions, such as, "Would your four best friends be a good way to select a representative sample? Why or why not?" She then involved the students in a "sampling" activity from the textbook in which they were to take five minutes and ask their classmates one of the questions, "Do you prefer bicycling or roller blading?" "Do you prefer to do homework by yourself or with friends?" or "Do you prefer to take part in sports events or watch?"

In this classroom example I could see Terry using teacher-directed talk to emphasize math terminology in her use of such terms as *line plots*, *biased sample*, *majority*, and *numerical*. She was using talk to encourage the students to think about their learning in her use of such questions as, "What conclusions can we draw from this simple line plot?" and "Could I say that all Grade 6 students watch less than two hours of TV a night?" She was also encouraging her students to think of real-life examples to make their learning more relevant because their assignment had been to keep track of how they spent their leisure time. Much of this talk was related to guiding the students, explaining the math textbook questions they had to complete. There was some student-initiated talk that

took place when the students were asking their classmates questions for the "sampling" activity.

The NCTM's (1989, 1991) *Standards* documents emphasize the importance of social interaction and communication in learning mathematics. The talk that took place in this first example was almost entirely teacher directed and didn't enhance the students' involvement in classroom discourse, which is vital to the formulation of understandings in mathematics.

January 7th and 8th Lessons

Terry had required the students to complete textbook pages on line plots for homework the night before. On January 7th Terry involved the class in a lesson about scatter plots versus line plots. The following is an example of some of the talk that transpired during this lesson:

> T: When would you use a *scatter plot versus* a *line plot*?
>
> S(1): *Scatter plots* are for heights. They're all different. On the *scatter plot* they're all different.

Terry then asked the students to open their textbooks to a page on line plots and guided the students through the questions on that page. She continued:

> T: About how many times do you hiccup before stopping?"
>
> S (1): B would be best [referring to the line plot that best represented this].
>
> T: Because?
>
> S (1): You don't usually hiccup just once.
>
> S (2): You could actually pick all (referring to line plots) because you can't control hiccups.
>
> S (3): Any amount usually up to 30.

At this point in the lesson the students were all giving different answers. Terry responded that there was not really a right answer. Terry continued talking about the textbook question that stated, "A Grade 6 class was surveyed. Look at the line plots on this and the next page. Each line plot shows the results for one of the five questions students were asked. Match each line plot to a question. Explain your reasoning."

Terry then involved the students in an activity in which they were to survey classmates about the length of time they spent brushing their teeth per day, the number of books each classmate thought that he or she would read in a year, and the number of chocolate cookies each classmate said that he or she could eat in a sitting. The students then took 20 minutes to ask their classmates questions. When they got back together as a class, the following talk transpired:

T: Any problems?

S(1): Some kids say they can eat 60 cookies.

T: You cannot judge; you simply collect and show it. Other problems?

S(2): Depends on how you ask it.

S(3): Sometimes when we went to other people we overlapped.

T: How could you remedy this?

S(2): Get a class list.

T: Any other problems?

S(4): Data collected didn't match the *line plot*.

T: What comes first? Should get your answers first, then plot. These are problems researchers find all the time. I am very pleased you had the wherewithal and could verbalize this.

On the following day Terry involved the students in a discussion about double bar graphs from page 161 of their textbooks:

T: Look at page 161. They've compared students in different grades and how they spend leisure time. From looking at a graph we can see where Grade 4s and 6s spend their leisure time. What activities are high for both?

[One student answered that TV watching is high for both.]

T: How many students were *surveyed*?

S(1): Fifty for each.

T: Describe how you got the answer.

[At this point I could not hear the student's response.]

T: You added.

S(1): Can't tell because the number isn't there.

S(2): Each box represents two students.

S(3): Certain amount of uneven people.

These lessons were portraying examples of teacher-directed talk and student-response talk. In the January 7th lesson, the talk that occurred after the students surveyed their classmates was more open-ended as they talked about dilemmas with respect to their data collection. I could see Terry using talk to encourage students' extension of their thinking by asking them a question such as how they would remedy the problem of the overlapping of responses.

Terry used words such as *describe* and *because* to support students in their explanations. I also noticed the continued emphasis on math terminology such as *scatter plots, line plot*, and *surveyed*.

January 14th Lesson

On January 14th the students were learning about stem and leaf plots. They opened their math books to page 164, and the following talk occurred:

T: Read through page 164. The last two days we've done double bar graphs. Some were missing important ingredients.

S (1): Need a title.

S (2): Need a scale.

T: Yes, you need a scale.

S(3): Label the axis.

T: Yes, you need to label.

S (1): Use a ruler.

T: Anything else you've learned? Yes, if you're missing labels. Stem and leaf plots, use it for information in double digits.

[Terry then put a T chart on the board with the headings "stem" and "leaf" and put 60 in the stem column and in the leaf column the decimals 0.1 ,0.2, and 0.3.]

S (4): Does it always have to point to something?

T: Good question. Look at number 8 on page 172. They ask you to plot in the range that's the lowest mark.

S (1): 58.

S (2): 97.

T: Do I have any other marks in the 50s and 60s?

The lesson continued, and Terry guided them through the work on page 172, where they had to make stem and leaf plots to show the marks of a fictitious Grade 6 class. In this lesson Terry, through teacher-directed talk, was emphasizing math vocabulary, guiding the students, and using representation to guide the students in their learning of stem and leaf plots.

January 26th Lesson

The students started work on their data-analysis projects in January. They each had to pose a question and survey a specific population of the school to answer their question. The students were encouraged to come up with a question that they wanted answered. In the days that followed the introduction of this project, the students were involved in going to other classrooms to collect data to answer their questions. The lessons after this involved clarifying for the students what they had to do, and during this time much of the talk centered on what the students were to do in their think books. An excerpt from the January 26th lesson is an example of the talk used to clarify the assignment:

> T: The notes you are writing to the other teachers, the teachers say they are funny. The notes don't say what time you want to go into their class or how long you need the class for. Have you experienced any frustrations?
>
> S (1): Whenever I went they were gone.
>
> S (2): Did you get any notes back?
>
> T: It may be a frustration for you. You may want to write in your notebook.
>
> S (3): The first time I went I only had half the class.
>
> T: You need to write this in your book.
>
> S (4): Does it matter where you write?

Terry was trying to guide and help the students overcome the frustrations they experienced with their data collection. The talk during this episode was more open-ended, but still involved teacher to student and student to teacher exchange. Students were using talk to reflect on mathematical situations that would have a direct bearing on the success of their data collection. I believe the nature of the assignment had a bearing on this as

there appeared to be more opportunities for students to use talk and writing to explore and discuss their mathematical work and investigations.

January 27th Lesson

On January 27th some students were going to other classrooms within the school to collect data, and others stayed in the Grade 6 classroom. I met individually with some of the students during this class and asked guiding questions to help them clarify their data questions, their survey population, and collection of the data. As part of this lesson, Terry asked more open-ended questions:

> T: I have questions I'd like to ask with regard to the think books. Hands up, who still needs to collect data?
>
> [Terry then wrote reflection questions for their think books on the board. Students were walking around asking classmates questions. One student asked me if I was doing a survey. I explained I was doing qualitative data collection, watching and writing down what I saw.]
>
> T: Are there any more questions?
>
> S (1): I don't understand question seven that says, "I wonder about . . . ."
>
> T: The more you know, the more you don't know.
>
> S (2): Do I have to answer why the question was important?

Some students were busy in conversation with their peers; others were doing their own writing. Robert asked me if he could use a decimal system to describe the data. I explained that I did not think that he could have 0.2 of a child. I asked Terry for clarification; she agreed that the decimal should be rounded up or down. Terry then gave out their unit tests and went over the correct answers.

The talk in this lesson was teacher directed but led to student responses that were unpredictable, given the nature of the project. The students were much more concerned

with writing responses from their data collection in their think books than being engaged in whole-class discussion with the teacher. Some students were asking their classmates questions to do with their research question as they were part of the sample group, others were busy writing in their think books, some were asking Terry questions, and some were meeting with me. Opportunities for students to discuss their data-collection procedures with other students were evident, but still the learning environment was teacher directed.

Discussion of Talk in the Analyzing Data and Graphs Unit

The preceding descriptions provide examples of the type of talk that took place in January as the students were involved in the Analyzing Data and Graphs unit. The talk in the first three scenarios was for the most part teacher directed, where Terry presented information, gave directions, and asked questions usually related to the textbook. Rittenhouse (2000) refers to this as the teacher-question–student-response–teacher-evaluation model. Because Terry's lessons followed the same format each day, I could see certain patterns with respect to the use of talk in this classroom. Talk was used to review the previous day's work, to explain new concepts in math, to clarify what the students were about to do, and to emphasize mathematics vocabulary. Spillane (2000) finds similar patterns in his research:

> The teacher posed questions and set exercises that did not invite students to reason about mathematical ideas and processes. Further, the manner in which she initiated discussion by defining correct and incorrect answers seemed to eliminate any opportunities for students to engage in mathematical inquiry. (p. 320)

The emphasis on procedure in the classroom is important to Terry because she wants to make sure that the students have a good grasp of what they are learning to provide a base for future learning. The students seemed familiar with the way talk was facilitated in the

math lessons. They had constructed norms about what talk was and how it was to be used in the mathematics classroom.

The talk in Terry's classroom followed an I-R-E discourse pattern. Hugh Mehan (1979) conducted a study in collaboration with Courtney Cazden that examines the organization of teacher-student interaction in an elementary classroom for one school year. Mehan describes the organizational sequence as "Initiation-Reply-Evaluation." Lessons are characterized as a sequence of eliciting information from the students. Each initiation is followed by a reply from the students. If a reply demanded appears in the next immediate turn of talk, then the result is a three-part sequence. If a reply does not immediately appear, teacher-student interaction continues across a number of turns. Nystrand (1997) expands upon Mehan's work and examines the role of classroom and other instructional discourse in learning in eighth- and ninth-grade literature classes over two years. He finds that most schooling continues to be based on a transmission and recitation model of communication in which teachers talk and students listen. The talk that occurred in Terry's classroom followed the same patterns as found in Mehan's and Nystrand's studies.

The acquisition of mathematics vocabulary is extremely important to Terry. Fawcett (1995), in his dissertation study, writes about a teacher who wanted to "nail down" definitions early so that the definitions could serve as a foundation for what was to come. Terry noticed that some of the students were struggling to use mathematical terms when they were trying to describe and explain mathematical solutions to her, and she became more and more concerned with their lack of familiarity with mathematical terms as the study progressed.

In the lessons on January 26th and 27th I noticed that the students were given more opportunities to talk with both the teacher and their peers. I believe that this was the result of their participation in a "project." The students seemed more interested, probably because the interaction provided an opportunity to pursue a question that interested them, and it was a change from the regular format of their mathematics classes. Torbe (1981), Barnes (1992), and Dewey (1938) remind us of the importance of relating learning to the students' interests and experiences and the importance of social interaction as part of this process.

Terry is aware of the emphasis she puts on teacher-directed classroom talk and the lack of opportunities for group work in her classroom. At the end of the data-analysis unit she commented:

> One thing that I have done for the most part since you started coming is teacher talk and listening; we've talked a lot. But it would be interesting when we come into a new unit with angles to give them a problem with angles and put them in groups of two or three. (Interview, Jan. 27, 1999)

As we engaged in other units of study, there was evidence that Terry was trying to place the students in group situations that facilitated more student-to-student talk. I believe that this is the result of my presence in the classroom and Terry paying closer attention to what the students were doing. Schifter and Goldsmith (1997) write that one of the factors contributing to teacher change is listening to and observing students as they work within the mathematics classroom.

Terry and I had not explored how talk was used in mathematics before this unit. We became more aware of talk and had more conversations about it as the study progressed. I had not wanted to explicitly zero in on processes of communication on my initial visits for fear that Terry might feel that she had to do things differently. Neither of

us had realized the extent to which teacher-directed talk was going to be such an integral part of the mathematics lessons. In the initial interviews we talked about her interpretations of the process of communication. She believes that talk is the essence of communication in mathematics: "I think for mathematics . . . it's talking about what they're doing, making sense of what they're doing, explaining to everybody else" (Interview, Nov. 14, 1998). Terry's practice did not reflect what she envisioned.

Writing in the Data-Analysis Unit

As part of the data-analysis unit we wanted the students to think of a question and a population they wanted to survey. We wanted them to write about what they were doing and thinking in a think book. The idea for the think books was not from the textbook. The think books took the form of learning logs, and the students used them to write reflections on and interpretations of a research question that involved data collection. Many of the specific outcomes listed for the statistics and probability strand from the WCP (Alberta Education, 1996) include the communication process, and this project fulfilled these specific outcomes.

Background to the Think-Book Writing Project

As I was completing my last observation session before the Christmas break, I asked Terry how she was feeling about our collaboration and my presence as a researcher in the classroom. I felt that I was not being "useful" enough and that this would prove to be a constant struggle for me throughout my research. It was difficult for Terry and me to plan every mathematics lesson together when I was coming to the classroom only three times a week. Terry indicated that it would be interesting and helpful to plan a few units together. I was elated and, at the same time, a little scared. Until this point we had not

been planning collaboratively. I wanted to feel that I was a co-contributor, and an important part of our initial discussions prior to the research centered upon the importance of collaboration. I realized that my initial desire to plan collaboratively with Terry was not always possible. Sometimes we planned and made decisions together, and other times these opportunities were not there.

We started brainstorming on what we might do with this unit. Because I was not sure of all of the mathematical concepts associated with the data-analysis unit, I suggested a data-analysis *project* might be a good idea. I thought that a student project in which the students posed their own questions about something they were curious about, determined their sample population, and decided on their methods of data collection, analysis, and presentation would be beneficial. I had suggested that perhaps the students keep a data-analysis think book or learning log. Coming from a language arts background, I believed that this could be one more way to incorporate the process of communication within mathematics class. The use of writing in the classroom was very important to me as an educator; it had been a focal point for some of my previous research in which I had looked at student writing in the context of social studies

I was a little uncomfortable in my suggestion of having mathematics think books. Was it my place to suggest the use of these in mathematics to facilitate more writing? What were my boundaries as a researcher? By January I was becoming an accepted member of the classroom community. In our initial conversations about the possibility of working together, Terry had been very concerned that all eyes would be on her, and as a result we talked a great deal about the necessity of working together to construct the

meaning of the process of communication in mathematics. The think-book writing project gave us the opportunity to plan together.

Rationale For Including Writing In Mathematics

Engaging the students in writing was important to us because Terry's class had not had the opportunity to do this in mathematics up to that point in the year. We both saw it as a way for us to experiment with using a strategy that was unfamiliar to us in the context of mathematics. Torbe (1981), Medway (1980), and Britton (1970) emphasize the importance of exploratory writing as a means for students' negotiation of meaning in the world. We chose the label of *think books* for the books in which the students would be doing this writing. D'Arcy (1989) writes that think books and journals provide the space for learners to reflect, rehearse, reshape, and redraft what they know. The term *think book* was chosen as a label, but as Fulwiler (1987) points out, there could have been other titles given to these. He writes:

> While some of us who assign these personal notebooks might argue about what they should be called—logs, learning logs, daybooks, think books, dialectical notebooks, field notebooks, diaries, whatever—we would not disagree about their purpose and value: writing helps our students learn things better and these notebooks provide a place in which to write informally yet systematically in order to seek, discover, speculate, and figure things out. (p. 9)

Fulwiler writes about the increased popularity of scholarly attention to informal writing; more specifically, journal writing. He states, "Recent research and scholarship suggest that the informal language of journals is too important to ignore" (p. 1). Writing in an informal way is a means by which students can explore using their expressive and exploratory language, which enables them to make sense of what they are learning. It was

our desire that the use of the think books would assist the students in their understandings of data analysis.

Setting up the Writing Project

In this section I discuss our initial considerations relating to the terminology, audience, and format of the think books.

In our conversation on January 7, Terry and I discussed the terminology we would use to label the students' think books:

> M: I'm trying to think now that I'm back at university, . . . is this a reflective journal, is it a learning log, is it a personal reflective note? I'm just trying to think what would be the best terminology, whether it would be a math *think* journal.
>
> T: You could make your own title and then you define it [referring to students]. You could call it even—a process journal, . . . and then you could have a whole bunch of different—you could reflect after. There would be times of reflection; there could be times of learning. . . . So I don't know why you couldn't—it seems silly to me to have it isolated, that this is a reflection journal and this is—I hopefully learn from my reflections.
>
> M: I'm thinking, because they're all individuals, that we all look at things differently, and that's why I hate to have this too regimented. You're right, just a process journal, because when I did that work in that Grade 3 class with dialectic journals, it was really hard for the kids to fall into slots. (Planning Meeting, Jan. 7, 1999)

We were trying to visualize how these reflective books would look and what they would be called. As we talked, I realized that we were trying to conceptualize the purpose and process of these. Neither of us had used learning logs, think books, or journals in mathematics before and had had little time to read about them in professional literature. I had done action research for my master's work with dialectic journals in a Grade 3 social studies class. Terry had used learning logs in science. The way we label or define the

writing that students do in our classrooms is important because it has a bearing on the way the writing will be conceptualized and the manner in which the teacher responds.

An important consideration with respect to exploratory writing is the question of "audience." To whom are the students writing when they are involved in writing in the classroom? Is their audience another student, their parents, or their teacher? We discussed the question of audience in the following conversation:

> M: Terry, the other thing we have to think is, who would be the audience for them? Would it be you and me, or is it another classmate? I guess that's what you and I would have to decide, who reads them, how often they'll be looked at, and who would comment.
>
> T: That's right. I want to know who's going to read what I write before I write it. (Planning Session, Jan. 7, 1999)

In the end, the sole audience for the students' think books was Terry and me. In retrospect we should have had the students read and respond to each others' journals, which would have been a powerful tool for helping them conceptualize their understandings and enhance their writing. We should have given them more opportunities to talk to each other about their ideas. I think one of our constraints was the time factor. Terry wanted the unit to be finished by the end of January, and we were uncertain as to how this whole writing project was going to unfold.

We had discussions about what should be included in the think books. My initial idea of what the students would include in the think books was reflected when I suggested:

> M: Maybe keep a writing log and just all the stages they've had to go through, and they almost have presentations at the end with a chart. . . . I'm just sort of brainstorming now. I was going to say almost like a reflective journal, where they sort of write down their problems with it or what they don't understand or their

> "Ahas" or "This worked great" or even "I'm having trouble getting the time to go into the other class." (Planning Session, Jan. 7, 1999)

Terry had suggested that the students should be taught how to use a reflective journal:

> T: I think that's a really good idea, but I think for kids, I think they almost have to be taught the reflective journal: What are you looking for in it? Maybe even modelled. (Planning Session, Jan. 7, 1999)

Terry demonstrated her acute knowledge of her students and knew that they had not experienced reflective writing in mathematics class before. My tendency was to give the students as much freedom as possible to see what their writing would "look like." Terry's tendency was to provide the students with more structure. This was very much a part of who Terry was as a teacher and reflected her philosophy. In conversation on January 14th we talked about the format of the think books:

> M: I was thinking we need one part for daily reflections on how they came up with the question on such and such a day and then the next day if they started to get their sample [they could address], what encounters or problems they may have had, other students' attitudes, or not enough time. I was thinking, like we were talking before, it's really important for them to be talking about these steps they're doing and, as you say, their data collection and analysis.
>
> T: This might be too unfocused. I was going to say to the kids that "your journal needs to include this, this, this, and this, and you organize it. . . . It needs to be organized in such a way that it makes sense to you . . . and that you can show someone the process that you've gone through." (Interview, Jan. 14, 1999)

The format of the think books was negotiated as the unit unfolded. We decided that we would discuss this together with the students during the next mathematics class. In our initial negotiations we were trying to conceptualize what the think books would look like and how their use would unfold in the classroom. As time went on we continually negotiated with the students and between ourselves about what we wanted and expected. I believe that we were not sure from the outset how these think books were

to look or be used. We were more focused on the means (the think book) rather than really focusing on exploratory writing as a vehicle for learning.

Implementation of Think Books

On January 19 we talked to the students about the data-analysis project and the type of writing we wanted them to do. I, for the first time, joined Terry at the front of the room. I asked the students what they did in their personal daily journals and explained that their think books would look different. I said that I wanted them to add a daily entry about questions, concerns, good things, and bad things about their data analysis. Terry wrote on the board and explained to the students that we wanted to see sections for the question, reflections, data-collection methods, data-analysis techniques, and a conclusion. Terry talked to the students about her research as a master's student and the conclusions she had reached. Terry's flair for providing the students with real-life examples was a trait that she knowingly pursued.

I believe that my role as researcher shifted on that day in the eyes of the students. I was no longer just the woman who watched and walked around helping students with their work; I was beginning to play a bigger role in this mathematics community. Not long after this the students started approaching me to guide them with some of their work.

The data-analysis project spanned three weeks. After the students were introduced to the think books, they tried to figure out the questions they would ask for the project. Some knew the question that they wanted to answer right away; others spent a great deal of time erasing questions and trying to brainstorm new ones. Nina wrote:

In grade one and six, how many like cats and dogs? (Field notes, Jan. 21, 1999)

Tait wrote:

> Which TV stations from Alberta are kids in years five and six most interested in? (Field notes, Jan. 21, 1999)

Kate's initial question was:

> Do you believe in evolution or creation? (Field notes, Jan. 21, 1999)

Believing it would be too difficult for her classmates to answer, she changed it to:

> Which flavour of ice cream do you prefer, chocolate, vanilla, or strawberry? (Field notes, Jan. 21, 1999)

Once the students had decided on their questions, they had to make their own arrangements to survey students in other classrooms. As the students started making entries in their think books, Terry and I had many conversations about the students' writing. We talked about their eagerness to get the writing done quickly, about the amount of structure to give, and about the quality of their reflections.

Concerns Related to the Implementation of the Think Books

Amount of Structure

As the writing assignment evolved, Terry and I kept coming back to the issue of the amount of structure to provide for the students. Terry felt that it was important to provide the students with guiding questions for their think books. In conversation she said, "Otherwise they all just want to run around and collect their data" (Interview, Jan. 21, 1999). At another point in the conversation she said:

> I think sometimes when we give kids an assignment without too many boundaries or enough boundaries, they just think and they don't even try. . . . I'm always disappointed in letting them choose, because I have in my head this neat little book with question and then a page for reflections.

Terry's use of the word *assignment* reflected her propensity for thinking of the exploratory writing as being structured in nature.

On the first day of the introduction of the data-analysis think books, I noticed Terry had written the following outline on the board:

Data Collection
- numbers
- reflecting
Data Analysis
- making sense and organizing
- graph, plot, and conclusion

Terry's need to provide the students with guidelines was integral to her way of teaching. We had not talked about this outline in our planning session, but I was always amazed at Terry's knowledge of curriculum and ability to see the "bigger picture." When I looked at the words on the board, they made perfect sense. Yes, this is what the students would be involved in as they thought through their projects. After the first few classes, Terry took in the students' think books to read what they had been writing. Noticing a lack of format, she wrote the following guidelines for the students' reflections on the board for the next day:

Reflections: Does this project seem like it will be easy, hard?
Why?
Did I understand what I was suppose to do?
Questions I may have at this point.

As part of the lesson on January 27th Terry wrote the following questions on the board:

1. What was the best part of this project?
2. If you were to do another project would you do the same type of question?
3. Would you choose the same sample group? Explain.
4. Could you use another graph to show your data?
5. Who would find your data useful?

As Terry was writing these, I came up to the board and asked if I could add a few. (I wanted to feel useful and that I was contributing.) I wrote:

6. What did I learn from this assignment that would be helpful to me in the future?
7. I wonder about?
8. I didn't understand _____ because . . . .
9. I liked/disliked doing this because . . . .
10. Why was the question I chose important to me?

I had not thought about guiding questions before our writing project began, and as it evolved I realized how important these questions were to become for Terry. I believe that these questions helped her maintain authority as she seemed uncomfortable with the uncertain outcome of the think-book project. We were both trying to figure out what these books should look like. Terry's desire to monitor her students and the work they produced was a foundational part of her teaching. I sometimes thought that the students were wondering, "What next?" with respect to the writing in the think books. We were always adding questions to help them focus their writing.

Terry's need for structure was an important part of this project, as it was for other assignments in which the students were engaged. I was not as concerned with the format of the think books, perhaps because I knew that I was not the students' teacher, and I

believe that my tendency as an educator leaned towards a less prescriptive approach to assignments. I had done all my teaching in the lower grades, and in my other subject areas I tried as much as possible to have students engaged with their peers in constructing knowledge. We talked about structure again near the end of the students' project as well as many other times during my research:

> M: Remember that first day we sort of said what we wanted included, and then we said, "Now, do this how you'd like"?
>
> T: And that, I think, was a mistake perhaps.
>
> M: I don't even think it was a mistake, but I know what you mean. They probably need more prescriptive, "You do this, you do this, and you do this."
>
> T: And perhaps even a timeline, because I found when I did social studies research, I gave them a very structured timeline, and they knew exactly they would have one day for this and two days for this and whatever. . . . So I'm not sure how you get around it, but I think being more prescriptive in how we wanted it written up would have helped. (Interview, Jan. 27, 1999)

Quality of Students' Writing

As the days progressed Terry expressed concern with the "quality" of the students' writing in the think books. She commented:

> I'm disappointed in their record keeping. . . . Some of them spent a lot more time drawing on the cover than actually doing the kind of record keeping that we would have wanted them to do. . . . In school they're so interested in the final project, and that's the frustration I found with them. (Interview, Jan. 27, 1999)

As I read over the students' samples of their writing in the think books after the conclusion of the study, I realized that many of them were engaging, honest, personalized accounts of what the students were doing. The students were writing descriptively about what they were doing at the time of the research. What follows are some examples of the students' writing and reflections in response to the teacher-directed questions: "Does this

project seem like it will be easy, hard? Why? Did I understand what I was supposed to do? Questions I may have at this point." One student wrote:

> Today was very interesting. When I went to Mrs. Smith's room and asked my question, all of them knew right off the bat which one they would pick. It took me 15 minutes in that classroom. It really went nice. When I went to Mrs. Langille's classroom things didn't go that well. The kids in the room were quite smaller and didn't know what they would pick. Many of them did that, about 3 kids said, "None of them." In that classroom it took me about 20 minutes because I had to explain more to them because they didn't understand it.

Another student wrote:

> I had to wait a while to collect the data from Mrs. Oren's class. I had to collect the data at recess and my pen froze. Lindsay was in the bathroom. I didn't have much time to collect data. It was worth going through all that because Mariah Carey won! It was hard to find some girls.

Another student wrote:

> This project was really fun. It was a good experience too. At the end of the project I learned what kind of thoughts go through a person's mind when they are doing something like this. My question has been answered. At the beginning I thought that A Channel would win and I was right.

Another student wrote:

> Day #1 Reflection:
>
> I will write Mrs. Wagner a letter. So far everything has been pretty easy.
>
> Jan. 25, 1999 Reflection:
>
> I asked Mrs. Wagner's class the question and they were there. I will also ask our class today, it may be my friends at recess.
>
> January 26, 1999 Reflection:
>
> Today I finally finished interviewing all the grade 6's. I began working on my title page.

These examples show that the students were responding to the guiding questions, were thinking about the logistics of what they were doing, and were personalizing their research.

As mentioned earlier, on January 27th Terry wrote guiding questions for the students on the board. The responses of some of the students to these questions follow:

Student 1

1. The best part of the project was seeing the results.
2. I would choose the same type of question because it was an easy format.
3. I would choose the same sample group because they were cooperative.
4. I think that the ice cream shops would find it useful, and so would people serving ice cream at parties.
5. You could use many different graphs like bar graph, double bar graph, line plot or pictograph.
6. I learned how to collect data and that in Grade 6 the most commonly liked ice cream flavour is chocolate and in grade one/two strawberry.
7. I wonder about why we did this unit in math because it wasn't very hard.
8. I didn't understand why everyone didn't say chocolate because it is very, very good.
9. I liked this because I was interested in what the results would be.
10. The question was important to me because I was anxious to see the results.

Student 2:

1. The best part of this project was collecting information and writing it down.
2. If I were to do another graph I would probably ask a different kind of question.
3. It would depend on the question as to what I would choose for my sample group.
4. I'm not sure who would find my data useful.
5. I learned that more of the grade 6 students thought there was life on other planets than grade ones. I also learned that grade one students could answer so easily and know what I was talking about.
6. I wonder what would happen if I asked a grade 3 class.
7. I completely understood the project and thought it was fun.
8. I like doing this because it's fun and interesting to hear different responses.
9. I'm not sure why I chose this, probably because it was the first thing that came to mind.

Discussion of Students' Writing in the Think Books

At the time of my research study Terry and I were somewhat disappointed in the students' think books. Later on as I reread the students' entries, I realized that many of their answers to the questions were of a personal nature. The entries mirrored honest reflections of what the students were experiencing in their projects. The entries were not lengthy, and I believe this was because of the structured questions we had them follow. Our questions elicited short answers. At the time of the research, I am not sure what we were expecting the students' writing to look like. The students were not used to

exploratory writing in mathematics, and I know that the only writing they had done in mathematics that year was writing in response to problems and writing definitions in their mathematics books. In retrospect, Terry seemed to be more disappointed than I was, because these were *her* students and she tended to react to their work a little more personally. I believe that she felt she had lost control to some degree over the outcome of the writing task and thought that somehow this was a reflection of her teaching ability. Our discussions at the time revolved around concerns with the students' struggles to communicate clearly about their mathematical thinking.

Nature of Teacher Feedback

Terry and I discussed the nature of the feedback we would provide in the think books. At the end of the third day of the data-analysis project, Terry read all of the students' think books and responded by writing clarifying questions on yellow stickies attached to the books. She believed it was very important for me to respond to each student's think book as well. We decided that I would confer with each of the students about their questions, how they were going to go about the survey, how the data would be recorded, and how they were going to write their reflections.

> T: And conclusions, maybe when we brainstorm we could ask, "What's the point of collecting this information?" Because if there is no point, why would you do it?
>
> M: Yes, and I think, too, that that goes with the conclusions but also the questions: "Why did you—why is this of interest for you?" It also might get them rethinking their questions too. If they say, "Well, I like ice cream," it might just get them thinking at another level: "Why do I want this answer? (Interview, Jan. 27, 1999)

On the fourth day of the data-analysis project, January 27th, I conferred one at a time with the students. I asked them about their data-analysis question and offered

suggestions on how they might go about collecting their data and writing about the experience. On this day I felt more a member of the classroom than ever before. The students were coming to me with their work, and I was responding to it. I wanted to become an accepted member of Terry's class in the eyes of the students, and thus I tried to give positive encouragement to the students: "Wow, that sounds great!" and "What will you do next?"

The think books were never graded but were used as ongoing assessment for that unit.

How to Teach Reflective Thinking

The WCP (Alberta Education, 1996) states that the study of mathematics should include opportunities to communicate so that students can "reflect on and clarify their own thinking about mathematical ideas and situations" (p. 6). Whitin and Whitin (2000) say:

> Writing and talking are ways that learners can make their mathematical thinking visible. Both writing and talking are tools for collaboration, discovery, and reflection. . . . Writing shares many of the qualities of talking, but it has some unique characteristics of its own, such as creating a record of our thinking that we can analyze and reflect upon. (pp. 2-3)

The question of teaching students how to write about their thought processes in mathematics was a concern throughout the study. Reflection did not seem to be part of the students' repertoire in mathematics, and Terry and I had many conversations about this. Terry had stated in response to my question regarding her perceptions of the students' written reflections:

> T: I perceive it was not well done, because I think they have been asked to "reflect upon," but reflection is different than answering questions. (Interview, Jan. 27, 1999)

I had similar questions with respect to the think books. In a field note entry I had written:

> I noticed these students did different things with these journals. Some are quite messy. I noticed the reflective part very scant or nonexistent. Maybe the students look at "communication" or more specifically "writing" differently in mathematics. (Field notes, Jan. 26, 1999)

We both noticed the students' tendency to rush through their writing. Terry felt that the students were almost too interested in the finished product. She wanted them to be cognizant of the process of their thinking in relation to mathematical understandings but was unsure of how to facilitate this:

> T: And maybe I'm not sure enough about teaching children how to reflect. I know that when they answer questions, the majority of them will do the fastest and quickest thing possible, because they see it as a task to be done. . . They don't really see it as learning maybe. (Interview, Jan. 27, 1999)

We had not had formal conversations beforehand about the modeling of questions and writing that would encourage thinking and writing about mathematical ideas and situations. As the project evolved, we realized how important talking about the thought processes is as a way to understand and express mathematical learning (Whitin & Whitin, 2000). In some of our interviews we talked about how we could encourage the students to start thinking about the process of mathematical thinking:

> M: Terry, when I came in today and saw them starting to work and the questions you had, the analysis topic seems to be going well, but did you have to encourage them some more?
>
> T: They don't seem to want to [reflect]; maybe they do need reflective questions and then think back on what they did do. But I want them to be thinking about process all along, and yet if they have never done it before, how will they know what the next step would be? (Interview, Jan. 27, 1999)

We did pose more questions, but these were not of the quality that encourages reflective thinking. Whitin and Whitin (2000) write about the importance of highlighting the

process by asking important questions such as, "How did you solve that problem? Who can explain that strategy in another way? Why did that make sense for you to solve the problem in that way? What other ways did people solve that problem?"

We talked about how group work would help enhance this reflection process:

> T: Maybe the next step is to get them to reflect in a group.
>
> M: Yes, that's right! I just thought of that.
>
> T: And it might be interesting to take a group of kids and you could lead it so they wouldn't go off on complete tangents and just see if the reflections were better when they could play off one another's ideas. (Interview, Jan. 27, 1999)

We did not do this, but through conversations such as this we began to articulate more explicitly how we might have set up the task so that the writing in the think books was more reflective in nature.

The students, Terry, and I were not used to writing that would encourage clarification of mathematical ideas. We all were struggling with how this was best facilitated and understood. Encouraging students to write as a means of clarifying their thinking about mathematical ideas is something that needs to be nurtured and modeled. In hindsight, I realize that we needed to reflect together, talk, and write together much more than we did. We should have had the students share much more of their writing.

Reflecting on the Student Writing in the Analyzing Data and Graphs Unit

As the data-analysis project came to a close, I realized that Terry and I had navigated uncharted territory with respect to the implementation of think books in Terry's mathematics classroom. What at first appeared to be a straightforward writing task opened my eyes to the complexities and uncertainties of writing in mathematics. Our deliberations about the terminology, audience, format, implementation, amount of

structure, quality of the student writing, and uncertainty of how to teach about writing as a way to clarify mathematical ideas were all critical aspects of the facilitation of the writing. Neither of us had done any preparatory reading or research about writing in mathematics, and all of our negotiations were a result of our day-to-day experiences with the writing project. We relied on our background experiences and hunches, finding security a great deal of the time in our traditional practices.

The format of the think books continued to shift throughout the project. Different headings and questions were added on an almost daily basis. These changes were a reflection of our uncertainty of how best to facilitate the students' responses. Terry was concerned about keeping the students on track with respect to the unit's expected outcomes, and her practice continued to be reflective of a transmission model of teaching.

The nature of our feedback to the students changed throughout the project, and I am sure that this led to confusion in the students' minds. I initially gave them individual feedback on their data questions and their methods of data collection, as well as encouragement on their writing. At the end of the project Terry collected the think books, and they became part of the document analysis used for this study. The books were never returned to the students.

Uncertainty about how to teach the students to reflect on and clarify their mathematical thinking was a recurring theme throughout the study. Before we can expect students to express their thoughts, it is critical that we involve them in many opportunities for talking with their peers. We were concerned about the "quality" of the students' writing. We somehow expected more insightful entries, but in read-backs of the

students' think books and in a closer analysis of our writing instructions, the students were indeed writing in response to our modeling and questions.

I believe that this snapshot of our writing project is an honest reflection of what other teachers may experience in the initial stages of incorporating writing into mathematics. Although educators such as Countryman (1992) and Whitin and Whitin (2000) give specific instructions on how to engage students in mathematical writing, at the time of our research neither Terry and I nor the students had had experiences with think books or reflective writing in mathematics.

Chapter Summary

In the data-analysis unit we were able to facilitate all of the specific outcomes that included the process of communication. We facilitated the students in specific outcome #1, the formulation of a question for possible investigation; specific outcome #2, the selection and use of appropriate methods of data collection; specific outcome #4, the selection and defense of the choice of an appropriate sample or population to be used to answer a question; specific outcome #6, the display of data by hand or computer in a variety of ways; and specific outcome #7, the reading and interpretation of graphs that are provided.

This was the first time during my research study that I had been in the class for the duration of a whole unit from the *Quest 2000* (Wortzman et al., 1997) textbook. I saw Terry using talk to introduce vocabulary, to guide and clarify concepts, to give instructions, to guide the students through textbook questions, and to review the previous day's work. Much of the talk was teacher directed, and Terry acknowledged an awareness of this in our interview session at the end of January.

As the study continued, the talk remained predominantly teacher directed, and the purposes for which it was used were much the same. Continued emphasis was put on mathematics vocabulary in the next few units.

I was very much involved in planning the writing project for this unit. The implementation of the data-analysis think books was a venture into unknown territory for the students and for Terry and me as teachers. Many questions were brought to the fore about how best to involve students in this type of writing. Terry and I began to examine our understandings of writing for learning and became aware of the complexity of involving Grade 6 students in writing in mathematics class.

We went on to involve the students in more writing in the geometry units, but the writing was not as in-depth and did not present as many deliberations for Terry and me as we had experienced in the data-analysis unit.

CHAPTER 6

FINDINGS FROM THE UNITS ON MEASURING AND ANALYZING ANGLES, EXPLORING GEOMETRIC SOLIDS, AND MEASURING AREA AND PERIMETER UNITS

Introduction

This chapter profiles the talk and writing that occurred in Terry's classroom during the study of the Shape and Space Strand that comprised Unit 8, Measuring and Analyzing Angles; Unit 9, Exploring Geometric Solids; and Unit 11, Measuring Area and Perimeter, from the *Quest 2000* (Wortzman et al., 1997) textbook (see Table 1).

Teacher-directed talk and the teaching of mathematics vocabulary continued to be emphasized during the instruction of these units. Terry continued her attempts to provide the students with opportunities for group work. Student writing emerged again as an emphasis in the Measuring Area and Perimeter Unit but was somewhat different from the writing that took place in the Data-Analysis Unit presented in Chapter 5. I present the findings from the units on Measuring and Analyzing Angles and Exploring Geometric Solids together. I then present the findings from the units on Measuring Area and Perimeter.

Emphasis on Vocabulary in the Unit on Measuring and Analyzing Angles

During the month of February, Terry started work on Unit 8, Measuring and Analyzing Angles; and Unit 9, Exploring Geometric Solids, from the *Quest 2000* (Wortzman et al., 1997) math series. The following are the specific learner outcomes and general learner outcomes from the WCP (Alberta Education, 1996) that were addressed by Terry. None of the outcomes emphasize the communication process.

Mathematical Processes:

C Communication	**PS** Problem Solving
CN Connections	**R** Reasoning
E Estimation and Mental Mathematics	**T** Technology
	V Visualization

Strand: Shape and Space (Measurement)

Students will:

- Describe and compare everyday phenomena, using either direct or indirect measurement.

General Outcome

Solve problems involving perimeter, area, surface area, volume and angle measurement.

Specific Outcomes

(# 2) Develop, verify, and use rules and expressions for the perimeter of polygons. [CN, PS, R]

(#4) Estimate and determine the surface area of a right rectangular prism, without using a formula. [E, PS]

(#6) Design and construct rectangles, given one or both of perimeter and area, using whole numbers. [PS, R]

[#10] Estimate and measure angles, using a circular protractor. [E]

[#11] Sketch and draw an angle when the degree measure is specified. [E,V]

[#12] Classify given angles as acute, right, obtuse, straight and reflex. [E]

These two units involved measuring and analyzing angles and exploring geometric solids. The students completed textbook activities related to measuring angles, building solids from nets, and finding volumes of cubes. I did not have as much input into these units of study as I had into the data-analysis unit. February was a fragmented month as far as the study was concerned. Terry was sick for a week, and teachers' convention was at the end of the month. The students were not involved in writing projects for these two units, and I continued to note a great deal of teacher-directed talk. However, I also noticed that Terry involved the students in more situations in which they were sitting in

groups. I noted an ever-increasing emphasis on math vocabulary and definitions, almost an urgency about it.

In this section I give examples of the emphasis Terry gave to vocabulary in these two *Quest 2000* (Wortzman et al., 1997) teaching units and provide a discussion using excerpts from various interviews. I go on to discuss the 3-D cityscape project. As part of the Exploring Geometric Solids Unit, the students had to create a three-dimensional cityscape with at least eight different shapes. I conclude by profiling the increase in group work that was becoming more apparent.

On February 18th the students sat in pairs making cubes from nets. As Terry walked around the room, she asked the students the following questions, all requiring the recall of factual information:

> T: A definition of an *edge* is what?
>
> S(1): It's the place where two *faces* meet.
>
> T: Can someone define *vertex* or *vertices*? What is a net?
>
> S(2): It's a two-dimensional representation of a shape.
>
> S(1): It's the representation of a three-dimensional shape.
>
> T: Yes, that's excellent; it's a *blueprint*. I asked you to do the net of a *cone*, *cylinder*, and *rectangular prism*.

At this point in the lesson the talk became a little less teacher directed. Terry said that she was going to add a fourth shape for them to create with a net, and this was a pyramid. She then drew a picture of a pyramid on the board and wrote:

"faces - 4"
"edges - 6"
"vertices - 4"

She asked, "How many faces? How many edges?" The students were then encouraged to share their nets with another person and try to construct a triangular pyramid.

Terry reiterated that the students had to think about *nets*, *cylinders*, *cones*, *rectangles*, and *prisms*. They continued to work in groups and talk about the shapes they had made. As Terry walked around the room, she encouraged the students to share ideas about the shapes they had made. She wanted them to share their nets with another person and try to construct a new net. She asked Andy why his shape had worked out, and he answered that it was because he had a graph. (Andy had been using graph paper for making his net.) Another student pointed out that the *circumference* had to be equal to the length. Terry then stated that the circles had to be *congruent*. She described the last task with which the students would be involved in that lesson: "You are going to work with the person next to you and you have to make a *cylinder* or *cone* that will work. (Field notes, Feb. 18, 1999).

In this lesson there was an extensive emphasis on vocabulary. Terry encouraged vocabulary responses from the students as they worked in their groups.

On the following day, February 19, I noticed a continued emphasis on vocabulary. The following teacher-directed instruction transpired:

> T: Boys and girls, there are a number of things I have to tell you. You need to know certain words: *radius*, *diameter*, and *circumference*. You need to understand these; for example, Kim is saying to Tom, "You have to measure 10 centimetres." Tom asks, "Like this?" [Terry gave an example of why students need to have the

vocabulary as a means to help describe their math learning to others.] "Tom doesn't know that Kim is talking about *diameter*."

T: We have to start using **vocabulary**. What's a *radius*? Yes, a *radius* is *half* the *diameter*.

Terry wrote on the board:

15 cm 10 cm

She then went on to say:

T: My next question is, what is the *circumference*?

S(1): It is the *perimeter* of the *circle*.

T: Yes, *circumference* is . . . .

Terry then wrote the algebraic equation for circumference and drew a triangle. She continued to use a teacher-directed approach and gave the children the formula for finding the circumference of a circle, which again limited student talk.

Reflecting on the Emphasis on Vocabulary

As the previous lesson ended, I realized that Terry was using mathematics vocabulary as a way to reinforce and explain math concepts. I reflected on my own memory of these definitions from my schooling with respect to the definition of some of these terms and realized I didn't remember some of these definitions. I decided to ask Terry about the emphasis on vocabulary in our conversation the next week. By that time the students were involved in creating 3-D cityscapes with geometric designs. I asked Terry to tell me about vocabulary in her math class. She responded:

> T: I think that through this project, it is going through a bit of a change. Initially it was just sort of, we're talking, and a lot of it was still teacher directed, asking questions and getting the kids to respond. And I'm finding now that if I present more of a problem, like "This is our project, this is what you have to do," and then I sit back and let them talk to each other. But something else I'm noticing is that I think we have to give them in a more structured way the vocabulary that they need, . . . because I've discovered in the last little while that they don't have the vocabulary. So when I started 3-D shapes we talked about, What is a face? What is an edge? What is a vertice? Before they got hooked into saying corners and edge—well, edges are edges—and sides and all of those things, I thought, No, let's start talking with the right vocabulary right away. But I have noticed in other things as well that they really do need much more vocabulary.
>
> M: For enhancement of their descriptions?
>
> T: Yes, Yes, we would often think about words in a lot of other subject areas, but you wouldn't think about it in math. (Interview, Feb. 24, 1999)

Terry was starting to think about talk in relation to other subjects. I told her that I had noticed her putting a lot of emphasis on mathematics vocabulary, and she answered that she had just read a book on science talk in which the children pose a question and talk it out. She continued:

> T: I was just thinking it would be kind of interesting to see if that would even work in math.
>
> M: Remember we were saying last week how the students find it difficult to think about how or why they've come up with something and sort of extend that one step further? (Interview, Feb. 24, 1999)

We never did try the idea. Terry was aware of the emphasis that she was putting on mathematics vocabulary for the students and saw their acquisition of vocabulary as a tool for their expression and understanding of their mathematics learning. Terry did not initiate student dialogue that would have given the students opportunities to explore their own understandings and definitions of mathematical concepts.

During this same conversation we talked about an instance on the previous day when we had noticed two students struggling for a term and a way to express their

mathematics learning. Travis and Robert were called to the board to describe, with the use of grids, how they had addressed the task: "You will draw a three-by-three grid. You have to find out how many squares in a four-by-four grid and a five-by-five grid. Find out a mathematical rule to find out how many squares in a six-by-six grid." The conversation of the two boys went as follows:

> T: In the three by three you have one big one, nine one-by-ones.
>
> R: You have the one, this one, one here, here, here, four of those.
>
> T: So the formula is, . . . if you have them in order, then you add numerals in order. (Interview, Feb. 24, 1999)

We could not help but notice that both students were not used to using conventional mathematics terminology to explain their problem solving. We talked about this:

> M: Today was interesting with Travis because he was saying something about putting—I think he was trying to say "numerals in order," but what other word was he saying? Do you remember that?
>
> T: Oh, yes!
>
> M: Oh, I have it here, Terry. I sort of thought that's interesting. I think that's math language, or how he believes the term is pronounced. I think he was calling it "numerals" or was it something else?
>
> T: He might have said that instead of "numerical order."
>
> M: It was interesting because I know too when students were sort of wondering how he was getting at that, he could write it, but he was finding it more difficult to explain.
>
> T: And I'm finding with the kids too it's sort of *my* putting words for them, and yet they brought up another idea that I hadn't thought about.
>
> M: That's right! That's right! That's communication in action again, isn't it! (Interview, Feb. 24, 1999)

Many times we noticed the students' struggles in their oral and written articulation of mathematics, and I know that this had a great bearing on Terry's continued

emphasis on vocabulary. We saw the students struggling with their writing and verbal explanations of what they were doing. I asked Terry how she conceptualized the process of communication and used it on a daily basis. She stressed that the students needed the "language" to communicate in mathematics:

> T: I've been thinking about this too as we've been working with it, and so far I've had them talk about what they understand or what they don't understand, but I think at times I haven't really taken the time to teach them the language that they need to communicate in math, . . . and just thinking about even the questions that they were asking in some cases. They use words like "and stuff," the very general kinds of language, . . . and sometimes they can't begin to articulate their own questions. (Interview, Jan. 21, 1999)

Terry commented in that same interview that perhaps the language we use in mathematics is somehow different:

> I think a lot of that is that we don't necessarily—I think the language in math is slightly different, and I'm thinking sort of on my feet here too, and it's something that's just been mulling about; we don't often use that language. (Interview, Jan. 21, 1999)

I think this realization that mathematics language is somehow different from other language led Terry to make a great effort to isolate the mathematical terms and emphasize them to her students. We were never really able to articulate exactly how mathematics language was different, but we talked about mathematics as being more conceptually oriented and not as narratively based as language arts. Terry continued the theme in the conversation, saying that we do not look at numbers as a way of telling a story:

> When I balance my cheque book, those numbers tell a story. I think math would become less removed from our real life if we would always look at it within that context as well.

I responded, saying:

> I believe that math language and the concepts we use in math aren't integrated through everything else they do. It's almost subject specific. It's so interesting, the books out on the narrative ways of knowing and knowing in math. I started to read a few of them, but it says our whole culture would have to start looking at it very differently. (Interview, Jan. 21, 1999)

We never did come back to the conversation of mathematics language being different, but I think our struggles with trying to implement and define the process of communication in mathematics were a result of what we had constructed through years of being taught mathematics in traditional ways.

An important aspect of Terry's desire for the students' acquisition of vocabulary is her desire for her students to become independent, autonomous thinkers. We talked about this:

> T: So I'm going, Wait a minute, maybe I'm doing these kids a disservice by assuming I know what they mean or by not making them articulate what they mean.
>
> M: I know exactly what you mean, and I think too, Terry, I remember even in the beginning how important it was for you as a teacher to make sure they were mature and responsible for looking after themselves, and now you're thinking, Can you really articulate without my help?
>
> T: And do you know enough to transfer that information into a new situation and be able to handle a new situation? It's not just, Can you solve these problems that we've done but what if you're in a new situation? Can you take learning that supposedly has happened and then move it into another one? I'm trying to think beyond this particular grade even. (Interview, Feb. 24, 1999)

An integral aspect of Terry's teaching philosophy is that she gives her students the tools to be able to apply their learning to other situations. She voiced this time and time again throughout the research study.

Terry also believes that vocabulary acquisition is important for "test" situations:

> T: So I'm listening and going, I know what you're saying, but I want you to tell me what you mean, instead of me providing them with language.
>
> M: Yeah, I know what you mean, and probably because you know you're not going to be there a month, or two months from now, to encourage that word or thought to come out of them. You're not going to be there.
>
> T: And what happens when they are writing a test, and I'm talking about a short answer? I may know that child well enough, and when I mark their short answer I may actually be thinking back to discussions that I've had with them and realize, Well, this is what they're meaning, but that's not what's down on paper. And so I'm going, Wait a minute. Maybe I'm doing these kids a bit of a disservice by assuming I know what they mean, or by not making them articulate what they mean. (Interview, Feb. 24, 1999)

Terry kept bringing up the importance of giving the students the tools with which to articulate their learning. Her desire for her students to be able to articulate mathematical ideas and concepts is very important to her. She believes in its importance because of the students' struggles in their oral and written articulation of mathematics, because of her desire for her students to become independent people, and because of the importance of vocabulary for written tests.

Terry believes that it is important that terms be clarified at the beginning of a unit of study. It means a great deal to her that the students be introduced to the definitions early so that they can build some of their mathematical understandings from them. I think that it is sometimes difficult for Terry to know when to let the students come to a knowledge and understanding of concepts on their own and when it is necessary to provide them with the definitions. She did not ask the students to articulate their own understanding of a concept before presenting them with a definition. The balance between allowing students to pursue their own ways of thinking and providing important information that supports the development of significant mathematics is not easy to achieve (Ball, 1993; Dewey, 1933; Hiebert, 1997; Lampert, 1991).

Emphasis on Student Talk in the Unit on Exploring Geometric Solids

In the latter part of February the students were involved in the creation of cityscapes. Their assignment was to create a city using three-dimensional shapes. It was actually the only project the students had within this unit. Terry's directions to the students were, "This is your math project. I'll be giving you nets; . . . you must create a city scene. Your 3-D cityscape must have at least eight different shapes. The 3-D shapes must be recognizable." This assignment spanned three class periods and gave the students the opportunity to work in groups. The two major opportunities for the group activities revolved around the construction of the cityscape and the assessment of the actual structure created. Since January I had been noticing that there were more opportunities given to the students to engage in work in which they were allowed to sit in groups. During the unit on geometric solids the students were sitting more frequently in groups and were more involved in hands-on activities and discussions as part of the lessons. This was a conscious effort on Terry's part, evidenced in the intention she voiced on January 27th: "One thing I have done for the most part since you've started coming is teacher talk and listening, but I think it would be interesting as we come into a new unit about angles to give them a problem and put them into groups of two or three."

Group Work During the Construction Phase of the 3-D Cityscape Project

An excerpt from my field notes describes the students as being very much involved in group work and discussion. On February 9th the students were sitting in groups varying in size from two to four children, as they had been for the most part since the beginning of January. As part of the introduction to the 3-D cityscape project, the

students had to estimate the measurement of the angles in an octagon, a triangle, and a rectangle. I wrote:

> One student is explaining the use of a "protractor." The students are explaining to Terry and me what they are doing. All are involved in discussion and trying to figure out their answers. Nadine and Todd explained to me what *obtuse* and *acute* meant, saying obtuse means going out and acute means going in more. Two boys at the table are using drawings to find out answers; two girls are using protractors. John explained to me, "I found out this is an obtuse and this is a sample. If I found out one degree, then I could find out the other." There are two groups of two students on the floor, one group of four talking, and another group of three talking and finding the measurement of blocks using protractors and guessing using 90 degree lines. (Field notes, Feb. 9, 1999)

I couldn't help but notice that the students seemed much more involved in their learning than they had been in previous lessons. This different structure appeared to enhance the opportunities for student-to-student talk. The students appeared relaxed, and the focus of their discussions was on measurement. Cestari (1998) writes, "Opportunities for children to reflect on their own understanding and reasoning about mathematics occur more frequently in situations in which they are allowed to express and clarify their mathematical thinking to others" (p. 170). The classroom structure was certainly facilitating opportunities for this to occur.

Group Work During the Assessment Phase of the 3-D Cityscape Project

Another example of an opportunity for student interaction in this project happened near the end of the project when the students were given the task of assessing each other's cityscape using a rubric that Terry had provided. The rubric was organized under four headings, Construction, Knowledge, Function, and Uniqueness. The students were asked to work with a partner and share in the marking of each other's city scene. I asked Terry about this:

> T: I was thinking about how am I going to assess the cityscapes. They took a fair amount of time, so I thought, there should be some kind of assessment that's a little more formal. . . . I've been working with rubrics, a fair number with the reports they've been doing in social and science. . . . What was my purpose for doing this activity? Obviously to learn the names of the shapes, so I thought that was one area we had to assess. I wanted them to put the shapes together in a way that was still recognizable, and I called that construction. . . . Then because I had put it within the context of a cityscape, I thought their buildings should have some function or purpose for being where they are located, . . . and some of them were actually quite creative, and other students then brought up, "What about creativity and uniqueness?"
>
> M: I think it was so neat, the categories that there were. And you're right, how they came up with the creativity and how you were so positive and said, "Yes, sure, we'll put that in." Have you used rubrics in math before?
>
> T: No.
>
> M: Because I was thinking, I hadn't either, and I too was used to using them in so many other subjects, and then as we talk, and so of course, why not? But it's still sort of how math looks different to us all. (Interview, Feb. 24, 1999)

It was obvious that Terry very much wanted the students to have experiences with assessment that would lead to more student ownership and discussion. It would have been very easy for Terry to have quickly evaluated the cityscapes on her own using a traditional marking scheme of a percentage mark, but she chose not to. She said, "It's much better, I think, than if I would sit down and look at each one. This way they could talk about; they could share." This experience also challenged Terry and me to think about both the assignment's purpose and what it was that was being evaluated. The following dialogue transpired as we contemplated this:

> T: There are people that would say, "This is a process that you're doing and you shouldn't be assessing process."
>
> M: When you say *process*, do you mean what the students did and what they came up with for the product?
>
> T: Yes.
>
> M: Okay, because you were assessing product and process, weren't you?

> T: I think it's okay.
>
> M: I think it's perfect, because I think of an essay done in an English class or an LA class, and it's the product, but we're looking at the process too.
>
> T: This way they could talk about, they could share, and a lot of them were really interested in saying, "This is this," . . . and they had good ideas.
>
> M: They did, and they looked very closely, and even Toss when we were looking at the edges and faces had an idea of what he should get, and I said, "What do you think?" He goes, "Well, this one here and this one here," and he said, "I'd give myself a four or a three." (Interview, Feb. 24, 1999)

We agreed that this activity had given the students the chance for interaction with each other. Terry wanted to give some ownership to the students for their assessment but wasn't ready to relinquish complete control. Even though she had created the rubric, she believed that she was giving them some responsibilities:

> T: I'm trying different kinds of ways to assess and evaluate. I think the kids are old enough, and I'm not trying to take away any of my ownership for the assessment and evaluation, but I think that if they have a hand in it too, then they become much more reflective as they're *doing* something.

I responded:

> M: I think that's true constructivism: They're taking responsibility for some of their own assessment, as you say, and just sort of learning to be more responsible too, Terry. And it's a nice age to do that. It's really hard sometimes when they're young, but this is a perfect age. (Interview, Feb. 24, 1999)

Through this project Terry had an opportunity to experiment with giving more ownership to the students by giving them some responsibility for the assessment of their projects. Terry was again trying to challenge her traditional assessment and instructional techniques. This had been a big step for Terry. The assessment phase of this project caused Terry and me to think again about the students' articulation of process versus product. We still were unable to express what exactly we wanted to see in the students'

writing or hear in their talk that would indicate that they understood the mathematical concepts. The question of how we could enhance the articulation of the students' mathematical thinking was always on our minds.

Reflecting on Student Talk Generated By Group Work

During the construction of the 3-D cityscape projects the students were involved in hands-on group work, both in the construction phase and in the assessment phase. We discussed the use of group work:

> M: I noticed that the students are doing a lot of group work in math. Can you talk a little about this?
>
> T: I don't know, I think with the problem solving very definitely, even if I had them sitting by themselves they would be discussing it with someone anyway. I guess if I kind of build discussion time in, then it kind of solves some of me saying, "Do it on your own" or "I want your work." I think sometimes when they're learning or just experimenting with an idea that they need to talk it out. After, on the tests, that certainly is independent work. I think one feeds off another person's ideas, so I'm finding that I'm doing much, much more group work. And part of that too is being aware of communication, of having them talk. (Interview, Feb. 24, 1999)

Terry is aware of the importance of student talk and that discussion helps students to understand mathematical ideas and processes. She also knows that this is best facilitated by group-work situations. She had mentioned very early on that talk was the essence of what communication in mathematics was about. Terry believes that talk is important in mathematics, but to create a classroom environment where this is practiced becomes a balance between what she is comfortable with and what she envisions as the optimal classroom environment that would facilitate student interaction and opportunities for communication. The increase in group work emerged slowly for Terry as she balanced her desire for a well-managed classroom with the desire for student interaction.

Each week I noticed the desks arranged in different configurations in the classroom. On my very first visit in November the students were seated in rows. After that I noticed almost weekly changes with respect to their seating arrangements. I wanted to ask Terry about this because I knew that giving the students more opportunities to sit in groups had not been an easy decision. We talked about this:

> M: Terry, they weren't sitting in groups in November, were they?
>
> T: No.
>
> M: Other years, do your students usually sit in a group situation, or does it depend on the year?
>
> T: It depends on the number of students I have and also the students. I like to have them sitting in groups, as long as they can work quietly and not bother others if they have to. I'm finding with this particular group they're not bad now.
>
> M: Was there an adjustment period?
>
> T: Oh, yes, around Christmas time and early January. I put them in groups and it lasted two days. And I thought, I can't handle this. So I moved completely away from it, and at the beginning of the year I had them sort of sitting in groups of two, but facing the board, all facing one way. But certainly they seem now to be coping better.
>
> M: They really do. I'm so impressed. I always just assumed that you did it all year long, and then I remembered that there were more rows. But no, I'm very impressed that they're on task and they seem to be doing just great. So, Terry it's sort of evolving as the year's gone on, what they do?
>
> T: Very, very much. This group has always been a talkative group. I'll say, "Line up for music," and they go over to their buddies and they start talking, and I just shake my head. It takes them five minutes to line up for music sometimes. We're getting better. What I was really afraid of initially and what did happen was they weren't talking about what they were supposed to be discussing; they were talking about anything else. But now I find, especially if it's a very structured kind of activity that they have to talk about or something they know they will be accountable for, then they stay on task pretty well. (Interview, Feb. 24, 1999)

Terry constantly balanced her desire for an interactive, constructivist mathematics class with her desire for control. She had slowly given her students more independence as the

year progressed. Initially, she found her students too talkative and was worried about their engagement in off-task behavior. Structured activities and some degree of accountability are important to Terry.

Through the 3-D cityscape project, as with the data-analysis project, chances for student interaction were made more frequent. The projects seemed to be a natural way to encourage this.

Discussion of the Units on Measuring and Analyzing Angles and Exploring Geometric Solids

During the units on Measuring and Analyzing Angles and Exploring Geometric Solids, the students were not engaged in as lengthy a project as in the data-analysis unit. The 3-D cityscape project was somewhat shorter in duration. During these units of study, I was again reminded of the importance that Terry gives to mathematics vocabulary. She feels its importance on many levels: as a tool to help students express and process their mathematics learning, as a way of providing students with mathematical understandings before a unit of study, as a way of helping them to become independent mathematicians, and as a tool for writing mathematics tests.

The 3-D cityscape project provided opportunities for student interaction and some degree of ownership for assessment through peer evaluation of the projects. Terry was making a conscious effort to involve the students in more group work, but the need for structure and accountability were always present. In this unit I noticed an increased emphasis on group work during both the project work and in the activities that followed the whole-group lessons.

Talk, for the most part, remained teacher directed. O'Conner (2000) calls this the "process-product" tradition of communication, saying, "Communication can be understood most simply as 'talk about mathematics' and 'learning' can be interpreted in terms of individuals' performance or assessment of their learning" (p. 18). The 3-D cityscape project facilitated student interaction and talk, and it became obvious as the study progressed that 'projects' were an avenue to facilitate more student interaction.

This unit again exemplifies Terry's struggle between teacher-directed learning and student-centered learning. Terry's dilemmas mirror other teachers' with respect to trying to implement the constructivist paradigm in mathematics (Cobb, 1994; Fennema & Nelson, 1997; Heaton, 2000; Schifter, 1996a; Thompson, 1991).

Terry engaged the students in specific outcome #2: developing, verifying, and using rules and expressions for the perimeter of polygons; specific outcome #4: estimating and determining the surface area of a right rectangular prism without using a formula; specific outcome #6: designing and constructing rectangles, given one or both of perimeter and area, using whole numbers; specific outcome #10: estimating and measuring angles, using a circular protractor; specific outcome #11, sketching and drawing an angle when the degree measure is specified; and specific outcome #12: classifying given angles as acute, right, obtuse, straight, and reflex. The communication process is not cited in any of these specific outcomes. The connections process, the problem-solving process, and the estimation and mental mathematics process are emphasized. In order to carry out these processes, the students had to be engaged in communicating, but it doesn't appear as an emphasis in the list of outcomes.

Emphasis on Teacher-Directed Talk and Vocabulary in the Unit on Area and Perimeter

During mid March and the first two weeks in April, the students were working on the unit on Measuring Area and Perimeter from the *Quest 2000* (Wortzman et al., 1997) textbook. Terry had planned this unit by herself. Because the unit was interrupted by the March break, Terry was becoming concerned about covering the curriculum in time for the provincial tests in May. The following chart represents the general and specific outcomes that Terry addressed in her classroom during the unit on Area and Perimeter, as found in the WCP (Alberta Education, 1996) under the Shape and Measurement strand. Once again the communication process is not referred to in any of the specific outcomes from the WCP (Alberta Education, 1996).

Mathematical Processes:

C Communication	**PS** Problem Solving
CN Connections	**R** Reasoning
E Estimation and Mental Mathematics	**T** Technology
	V Visualization

Strand: Shape and Space (Measurement)

Students will:

- describe and compare everyday phenomena, using either direct or indirect measurement

General Outcome

Solve problems involving perimeter, area, surface area, volume and angle measurement.

Specific Outcomes

[#6] Design and construct rectangles, given one or both of perimeter and area, using whole numbers.[PS, R]

[#7] Demonstrate concretely, pictorially, symbolically that many rectangles are possible for a given perimeter or a given area. [CN, R]

In this section I share examples of the continued emphasis on vocabulary, and I explore the writing and the talk that emerged as a result of the school plan project.

Terry started the unit by engaging the students in a discussion about the pictures in the beginning pages of the area and perimeter unit from the *Quest 2000* (Wortzman et al., 1997) textbook. There are pictures of a basketball court, a tape measure, a stadium, a swimming pool, a ruler, and various shapes such as a triangle and a square. The conversation went as follows:

> T: Tell me why there are these pictures.
>
> S (1): You could measure how far people swim.
>
> S (2): Tape measure so you know how many centimetres.
>
> S (3): Basketball place.
>
> T: What are the reasons for the polygon pictures? What would you know about area and perimeter?
>
> S (1): It has to do with sports.
>
> S (2): Has to do about measuring.
>
> T: Has something to do with shapes. We'll find out a little bit more.

The students' attention was then drawn to the next page, where there is a picture of a blueprint of a house floor plan. The questions and answers continued:

> T: What room do you see that you know? [All the students pointed and said something different.]
>
> T: Good. What does that tell us about measuring?
>
> S(1): You can tell about doors and windows. What about the place that has no in and out?
>
> T: Good question. What do you think it is?

S(1): Maybe where pipes and wires go to get upstairs.

T: Maybe a storage room. We'll continue talking after recess.

After recess Terry led the class in the correction of homework and then started talking about a project on creating a plan for a school. Thus began the unit on Measuring Area and Perimeter. I noticed in this introduction that Terry was using teacher-directed questions to guide the students to start thinking about area and perimeter. There was again a great emphasis put on the use of the textbook, which was in keeping with the usual profile of the lessons.

Writing in the Unit on Area and Perimeter

As part of the unit on area and perimeter, the students became involved in a project on creating a school plan. Terry introduced the students to the school plan project near the end of the first class on area and perimeter as described above. I did not know beforehand that Terry was going to give the students this assignment. They were to design the school of their dreams and include everything they thought the ideal school should have. As part of the guidelines, the school had to be situated on one hectare of land and had to include a minimum of six classrooms, a regulation-size gym, washrooms, and offices. Each classroom had to have an area of 60 metres squared. Terry's directions to the students were, "Since Mrs. Ellis is here, I want to give you this for tomorrow. You must draw a floor plan of a school and a playground. You must write and show how you used perimeter and area" (Field notes, Mar. 18, 1999). Terry then wrote on the board:

Criteria: One hectare is the area.
School and school yard must cover an area of 1 hectare (10 000 m^2)

- done to scale
- elementary
- 350-500 students
- floor plan of the school

Communicate in writing how you used perimeter and area.

Terry strongly emphasized the words *communicate* and *writing* in her directions to the students as the class continued that day. I was excited that the students would once again be engaged in an assignment in which they would be writing. At this point I didn't know what format the students would be using for their writing. I was curious about where this idea had come from, and Terry told me it was from their textbook, *Quest 2000* (Wortzman et al., 1997). In conversation a few weeks later, Terry said, "It was a project that was set up in the math textbook itself, and it had an assessment rubric with it, and I thought, well, here's something that's already done; why not give it a whirl?" (Interview, Apr. 14, 1999)

After Terry finished presenting the directions to the students, they asked the following questions:

S (1): Can we have more than one floor?

S (2): Can we have a playground and bike racks?

T: Whatever scale is up to you. One box is 10 metres squared if you want. Yes, some can be grass and trees. You must communicate in writing how you used area and perimeter.

S(3): Can we have a partner?

T: Everyone has to work on their own, but you can talk while you're doing it.

Terry and I were both curious to see what type of writing the children would generate for this project. I wrote in my field notes: "Interesting the focus on writing that will be asked for this assignment. We have talked that it really helps to let us see who understands and what they're thinking as they do it" (Field notes, Mar. 18, 1999)

On March 23rd and 25th the students were involved in creating their school plans and writing on the back of the plans about how they used area and perimeter. The classroom was a flurry of activity; some students were calculating their measurements, others were involved in drawing, and others in writing about what they had done. My field notes state:

> I walked around the room helping the students with their school mapping projects. They had to map an area of 10,000 square metres. Some students were still working on figuring out how many buildings they would have and of what sizes. Others had mapped it. Mark and Andy were finished, and Terry and I were coaxing them to write about how they used perimeter and area. Andy and Mark found it difficult to do; they kept coming back and saying, "Is this okay?" Toss and another student were working together. Some students were being silly, but most were on task, some talking, some not. (Field notes, Mar. 23, 1999)

Reflecting on the Student Writing in the Area and Perimeter Unit

As I looked at the students' work I noticed that many of the written entries on the back of their school plans were short explanations of what they had done, whereas others were more elaborate and reflected more in-depth knowledge of area and perimeter. I copied some of the students' writing in my field notes. Jenny wrote, "The way I used air and primeter is like this. I drow it and then I use a different colore and I times them but sometimes I used different color. 1 squer is 10 cm^2" (Field notes, Mar. 23, 1999; Sample 1). This sample reflects what she knew about area, that area has something to do with squared units.

Kerri wrote, "Need to measure perimeter and length, how big classrooms should be. Need to fit in all you need and some things you want without going over the limit. Measure right" (Field notes, Mar. 25, 1999; Sample 2). In this sample we see that Kerri saw a distinction between length and perimeter, but it didn't suggest that she understood perimeter as the distance around a figure.

Andy wrote:

> I just did what was on my paper except added a bit more. I needed to know that every square was 10 metres squared and the perimeter to put them together. Area means it covers the surface of a polygon or shape. The perimeter means the whole way around. The area means how much rooms there are in my school. It can hold 360 students in it. (Field notes, Mar. 25, 1999; Sample 3)

In this sample there is evidence of Andy's understanding of area as a count of identical squares that cover surface and perimeter as distance around.

Tony wrote, "I used area by knowing how many square meters I used. I used perimeter by knowing how much space I had and using my one hectare" (Field notes, Mar. 25, 1999; Sample 4). This sample suggests that Tony understood area as a count of identical squares because of the use of the phrase *square metres*, but he was perhaps a little confused about perimeter.

Tom wrote:

> I put something somewhere and if it worked out I would check it. I also used the squares. I looked at the classroom and determined if something needed to be bigger than the classroom or smaller. Since the classroom is ten squares it was easy to determine parts of the school because I took the ten and added or subtracted squares to get my school. I had to take four top rows off to get my hectare. (Field notes, Mar. 25, 1999; Sample 5)

In this sample, again there is evidence that the student knew how to count squares to determine area.

From close examination of these samples I believe there is evidence of the students being somewhat hampered by terminology because they had not had the opportunity to talk about and create their own concepts in relation to the definitions of area and perimeter. The WCP document states, "It is not enough to arrive at an answer. Students must be able to communicate effectively how the answer was obtained" (p. 6). I believe that Terry and I should have supported an environment in which students were encouraged to talk to each other about the concepts of area and perimeter. Their writing may have reflected a much different interpretation and understanding of these concepts had we done this.

At the time of the research Terry and I had both expressed disappointment with the students' written responses. We had expected longer entries, a more thorough understanding of area and perimeter, and a better understanding of how the students arrived at their solutions. I realize that we had not really modeled what the written responses might look like because we too were unsure of how to effectively teach the students to become cognizant of their thinking about mathematical ideas. The task had been taken directly from the textbook and did not leave much room for interpretation. As I reread the think-book samples months later, I realized that the students had indeed responded as best they could to the writing task. My field notes from March 25th reflect that Terry and I were concerned about how best we could facilitate the students' writing. My field notes read:

> Students are talking as they're completing their work. Some are using calculators, some just counting. Mark and Andy are finished. We really have to encourage the children to write about their learning, about *how* they used area and perimeter. Terry believes they have lack of math language with which to discuss and write how they are doing math. She also believes it's a lack of discipline. (Field notes, Mar. 25, 1999)

In the April 14th interview I asked Terry how she felt about the students' writing for this project. She answered:

> Terrible, for the most part. And you know, I think they had so much fun just drawing the school that they didn't want—not that they didn't want—they completely forgot about other criteria. . . . I think that the whole project, as far as I'm concerned, was a bomb. (Interview, Apr. 14, 1999)

I tried to encourage Terry by saying that I felt that the students had been very involved in the project, but at the same time I too was concerned about the quality of their writing, expecting more explanation and emphasis about how they arrived at their explanations for area and perimeter. Yet I now realize that we had not provided an environment conducive to exploratory writing.

We had not modeled what we wanted the students to do in their writing, but we did help them one-on-one with their writing. Another excerpt from my March 25th field notes says, "Vince came up to Terry and me, and we gently encouraged him to write about how he used perimeter and area. He came back twice as he was confused about what to do; he didn't understand what he was to write." I believe that this statement reflects the difficulties that other students may have been having with this task as well.

Terry was concerned not only about their writing in the project, but also about their mathematical knowledge about measurement. She explained:

> But even in the mathematical part, the thinking, I just felt because they were just saying, Well, I'm going—like, they didn't have any real concept of space and what size this room would be and how big you would have to have for 30 students. And, I mean, somebody had an ice rink. They were going to have an ice rink in their school, and they made it about 20 square metres, you know. I said to them, "Now!" They thought that was plenty big, and then we started talking about it, so it told me that their real-life experience with measurement was almost nonexistent. (Interview, Apr. 14, 1999)

Terry believed that their real-life experiences with measurement were lacking. However, writing had helped us to understand what the students had understood and not understood. We discussed this:

> M: So, I think it's, you know, it's good we had that writing component because we probably would never have known what they were learning.
>
> T: That's very true. I think I knew then that they didn't know what I thought they knew. (Interview, Apr. 14, 1999)

The students had not done well in their unit tests on perimeter and area at the end of the unit, and Terry and I both surmised that the writing on the school plan projects had been an indication of what the students did and did not understand about the concepts. We discussed their performance on their tests:

> M: Even in those tests, they didn't.
>
> T: They didn't know what they were doing either. I think the writing—I sort of thought, okay, they're having troubles expressing themselves because they weren't expressing correctly what I assumed they know. But then, I guess, I did take it one step forward; they really did not know it anyway. (Interview, Apr. 14, 1999)

We also discussed the fact that Terry had talked to the students about area and perimeter, but this had not translated into success on the tests:

> M: You went over the unit test last week on perimeter and area, and why do you think the students performed the way they did?
>
> T: I think part of me was assuming that they knew what area was and what perimeter was.
>
> M: Yeah, but you had talked enough about it.
>
> T: I did. I had about 10 kids that got it and the rest of them, they just tune you out. I think this could have been what happened. They pay attention if I say to them, "You're going to need this and this and this."
>
> M: Right.

> T: "So now I'm going to tell you about, or give you some clues or whatever as to how you will need to do it, because if I just start talking about something you'll look around," and some of them will be here, there, wherever; and you bring them back for a little bit, and they're kind of gone again. To be fair, this was the week before spring break. (Interview, Apr. 14, 1999)

As I reflect, I realize that the students were not learning from Terry's teacher-directed talk about area and perimeter. I believe that the students were getting tired; spring break was fast approaching. I know that the students would have benefited from much more participation in defining mathematical ideas and sharing of their writing. The students have to be actively involved in clarifying their thinking processes about mathematical ideas and in the defining of concepts for them to have personal significance. Their performance on the tests was a testimony to this.

The students had fun creating their school plans. The details in their maps were fascinating. They had everything from antigravity rooms to scuba rooms to bumper-car rooms to rock-climbing rooms included in their maps. I believe that the students did learn something about measurement in this assignment, as evidenced in the work samples collected. Perhaps it did not translate into knowledge on the test because the test was not project oriented and had basic single-response questions.

The students were involved in a different type of writing for their school plans than in their data-analysis projects. In writing about their school plans they were attempting to write more about the process and strategies used, whereas the writing in the data-analysis projects had been in the form of documenting what they had done. We had not modeled the type of writing we expected for this project. Their writing for the school projects was short and direct, as were our instructions.

I believe that Terry was always concerned that she had not tried hard enough and that her students were not doing as well as she wanted them to do. This was an ongoing feeling that I, as the researcher, tried hard to dispel. I wanted to boost her confidence because I believed that she was working as hard as she could for her students. The struggle would continue for Terry.

The students had no previous experiences with reflective writing in mathematics. They were not used to writing about how they came up with solutions. We were not used to writing about this ourselves. As I reflect now, I realize that I do not always understand my thought processes when I am thinking about mathematics. Getting the students to clarify and express their own thinking about mathematical ideas was always a worry for us. It came up time and time again with respect to the students' articulation in their written and oral communication. Terry, the students, and I were all struggling to a degree with this. How could we encourage the students to reflect on process? How were we constructing the notion of process in mathematics? Did we even know enough about it?

A big question for me is why we were disappointed in the students' written entries. Was it because we felt they must be perfect because they were part of a research project, or did we think we were failures as mathematics teachers? I believe we were starting to realize that we too had much to learn about facilitating writing in a mathematics classroom. Was it more the results on the tests that were bothering Terry? Terry had told students while passing back the tests, " You either did really good or really bad" (Field notes, April 6, 1999). In conversation with Terry later that day, she said, "They didn't know what they were doing [on the tests]" (Interview, Apr. 7, 1999). I believe that the writing in response to the question of how they came up with area and

perimeter was probably frustrating for the students. They had not been given the opportunities to explore their own mathematical definitions with respect to these concepts and as a result were hindered in their ability to explain what they were doing. There was a great deal of talk as the students created their school plans, and perhaps a different writing task would have allowed the students to express more authentically what they were thinking.

Student Talk in the Unit on Area and Perimeter

The students were engaged in a lot of talking during their school plan projects. It was during this project that I noticed the students were more consistently engaged in student-to-student talk. Terry and I discussed the student talk in this project a few weeks later:

> T: I found that the kids, even though they weren't actually sitting, you know, beside someone, they were very much talking about their school.
>
> M: I noticed that too.
>
> T: It was a noisy activity. Lots of dialogue, lots of discussion. And it was kind of interesting because one of them wanted to have waterways instead of hallways, and so your classroom would be mobile, and you would have the music room that was stationary, and the gym. But if you were going to music, you would just chug your little classroom down the waterway, and latch up at the music room, and then go in there. (Interview, Apr. 14, 1999)

We agreed that the students seemed to enjoy working on the school planning projects. As we continued talking, Terry pointed out that if the students had to describe how they did on this project, they probably would not talk about area or perimeter, but about why they had certain things in their school. I believe that she touched on a key point with respect to the students' mathematical projects. Our conversation continued:

> M: That's true, and probably that's very important for us to know too, isn't it, as teachers? I don't know in all subjects what is really of interest. It's probably their real life, the constructing, you know.
>
> T: Well, and they've been in school for six years, probably sitting there wishing they had A, B, C, you know, and whatever it was that they're interested in. Like, a science room would be wonderful, and that's one thing I would love to have, or an art room, or even a sink in my room with water. I mean, so the kids, then are thinking, Hm, and you can tell one thing that it does point out. This is probably not mathematical, but it's important to the kids. (Interview, Apr. 14, 1999)

We had our own ideas with respect to what we believed was necessary in Terry's classroom for optimal mathematical learning to occur. At the same time the students were experiencing mathematics in that classroom in their own unique ways. The students' perceptions of their school planning projects and the mathematical understandings varied from student to student. We had noticed the students highly engaged in talking about the special attributes their schools would have. Some schools had TV rooms and pools, and others had movie theatres, go-cart racing, and Sea-Do and scuba-diving rentals. I don't believe that the students were necessarily thinking about area and perimeter and how they calculated these at a totally conscious level, but they were still very much involved in measuring nonetheless. We realized that through this project the students were engaging in much desired talk, both from their standpoint and ours. I began to realize that it was through these projects that Terry was encouraging the students to engage in the talk, writing, and group work that facilitated the process of communication.

Discussion of the Unit on Area and Perimeter

As the unit on Area and Perimeter was nearing an end, I realized that we had again experienced both frustrations and key understandings with respect to facilitating the process of communication in Terry's mathematics classroom. We continued to struggle with the quality of the students' writing. In retrospect I realize that we had not modeled

the type of writing we expected and we were not sure what we were expecting. We did not have preconceived ideas about the type of writing we wanted. As we read the students' entries, we realized that the students had found it difficult to write about their mathematical thinking and describe in depth the concepts of area and perimeter. Asking questions and engaging in conversations with others would have facilitated the students' ability to talk and write about how they calculated area and perimeter. Perhaps a different writing task would have given the students a better opportunity to express what they had learned and given us a better indicator about what they were learning. The students would have benefited from writing in mathematics in all of their years of elementary schooling. We felt that the students' writing was an indicator of what they were conceptualizing about area and perimeter, and Terry questioned her teaching as a result of the performance on the tests.

At the beginning of this unit Terry again engaged the students in teacher-directed questioning, but the school plan project at the end of the unit gave the students opportunities for student-to-student talk. This was the last project completed by the students during my study, and in this unit I had noticed increased opportunities for student talk and group work. It appeared that Terry was still struggling to become comfortable with this approach to instruction, and in the unit that followed, opportunities for group learning seemed to diminish.

The students were involved in specific outcome #6, designing and constructing rectangles, given one or both of perimeter and area using whole numbers; and in specific outcome #7, demonstrating concretely, pictorially, and symbolically that many rectangles are possible for a given perimeter or a given area. The students were involved in the

process of communication; but, interestingly, the communication process was not listed in the WCP as one of the mathematical processes emphasized in order to meet these outcomes.

Chapter Summary

As Terry and I moved through units 8, 9, and 11, we again experienced uncertainties with respect to facilitating the process of communication. Talk was predominantly teacher directed, with continued attempts to facilitate more student interaction and talk. Terry used talk for the purposes of guiding learning and encouraging clarification and explanation of new concepts. Terry and I tried to introduce opportunities for student-centered talk by introducing the 3-D cityscape project and the school plan project. The students were sitting in groups as they worked with these projects, and this facilitated a little more student interaction and student-oriented talk.

We engaged the students in writing about how they determined area and perimeter in their school plans. This idea for this came from the *Quest 2000* (Wortzman et al., 1997) textbook. We were not happy with the students' writing but in retrospect realize that the task did not lend itself to exploratory writing.

Uncertainty and confusion about how to teach the students to value the process of mathematical thinking was a prominent part of our discussions. Mathematics educators know that through facilitating questioning and encouraging students to talk and write with each other about their mathematical deliberations, their understandings of the process of mathematical thinking will be enhanced (Whitin & Whitin, 2000). At the time of the study Terry and I were still struggling with how best to facilitate this.

The emphasis on mathematics vocabulary was important to Terry. She saw this as a way to ensure that the students would have the terminology to use in test situations, and she believed that it helped them to better conceptualize their learning. Their explanations in their writing about area and perimeter reveal that they did not have enough opportunities for engagement in their own defining of mathematics terminology.

Terry was challenging the safety of her traditional practice and attempting to facilitate student-oriented approaches to mathematics in her classroom. Her growth was not linear. I found at times that Terry had structured the learning environment to be constructivist in nature, and other times there was a great deal of teacher-directed instruction. This seemed to fluctuate along a continuum that could be described as a transmission (teacher-directed) -interpretation (student-oriented) continuum (Barnes, 1986). Terry felt safest at the transmission end.

CHAPTER 7

CONCLUSIONS AND IMPLICATIONS

> I think some days my teaching style does not reflect my philosophy. . . . Often times the stories were not reflective of what I said my beliefs were. That causes great angst to a teacher. (Interview, May 5, 1999)

In this chapter I examine the many contextual variables that impacted Terry's interpretation and implementation of the process of communication in mathematics, and I raise questions in relation to the views of constructivism found in supporting mathematics documents. I conclude by discussing the implications derived from this study and make suggestions for further research.

Overview of the Study

I began this study with the research question, "How does one teacher interpret and put into practice the process of communication as identified in the WCP document?"

The participants for the study were Terry and her class of 23 Grade 6 students. Data-collection procedures utilized in this study included writing reflective and descriptive field notes, participant observation, interviews with the teacher, and collecting documents. The data sources were analyzed by identifying common themes that emerged within and across data sources.

The overriding theme that emerged from the study was the powerful influence of the contextual variables that affected Terry's interpretation and implementation of the process of communication in mathematics. Terry is an experienced teacher who is knowledgeable about mathematics and holds a constructivist philosophy. Yet the findings of this research suggest that it is extremely difficult for her to actualize practice consistent

with her knowledge base. The reasons for the tensions and fluctuations with respect to implementing a constructivist approach towards the process of communication in Terry's class are complex.

In this chapter I will revisit and discuss each of the contextual variables that impacted Terry's interpretation and implementation of the process of communication.

Exploring the Relationship Between Contexts and Terry's Practice

In Chapter 4, I initially discussed the many contextual considerations that influence Terry's instructional decisions. In the final analysis, these contextual variables not only remain crucial components of Terry's teaching world, but in many ways actually work against the implementation of the process of communication in her mathematics teaching. In this section I isolate those contexts that appear to have the most bearing on Terry's interpretation and implementation of the process of communication. Some mathematics educators look at the tensions created as a result of the differing and oft times conflicting contexts as an integral component of teacher change. Jones (1997) asserts that there are many faces of context that are important in helping to understand teacher practice. He emphasizes that the tensions and responses to the tensions in varying contexts, such as changes in teaching methods, what counts as curriculum content, and changes in epistemology, can work together to bring about teacher change. He states, " I have argued that interaction with, and response to, tension play a big part in teachers' change" (p. 147).

Culture of the Classroom and School

The culture of Terry's mathematics classroom has an impact on her instructional decisions. I am aware that there was a mathematics classroom culture in place prior to my arrival in the classroom, but on the days I was there, I too became part of the culture. Terry, the students, and I all brought embedded expectations from years of constructing meaning about the teaching and learning of mathematics. The ways in which Terry and I interpreted and facilitated mathematics instruction and the ways in which the students responded to and shared their understandings of mathematics were part of the social norms (Yackel et al., 1991) of this classroom community.

The social norms are integrally linked to how the process of communication is enacted in Terry's classroom. There is a generally accepted protocol in Terry's mathematics classroom, a distinct way in which she teaches, and a distinct way the students are expected to engage in mathematics learning. Spillane (2000) finds the same tendencies in his study of a teacher's practice in mathematics. In his study the teacher approaches mathematics instruction in a traditional manner, with a reliance on the mathematics textbook and a concentration on computational skills. The pattern of the classroom discourse in Spillane's study consists of the teacher reading equations from the textbook while the students compute the answers. Some students are chosen to verbally provide the answers and justify them. The discourse patterns, Terry's questioning techniques, the way the students were expected to respond, the students' familiarity with the classroom agenda, the accepted protocol for working in mathematics class, and the generally accepted protocol for working in groups constitute normative behavior within

the culture of Terry's classroom. Terry approaches mathematics in a structured, traditional manner. Goldsmith and Schifter (1997) define traditional practice as follows:

> Traditional mathematics instruction is grounded in the belief that students learn by receiving clear, comprehensible, and correct information about mathematical procedures and by having the opportunity to consolidate, automatize, and generalize the information they have received by practicing the demonstrated procedures. The teacher and text provide the source of this information, and hence have the mathematical authority for determining right and wrong, "good" and "bad" mathematics. Classroom instruction is organized around the transfer of information from knowledgeable teacher to uninformed student. (p. 22)

In Terry's classroom there was a teacher-directed question-and-answer period at the beginning of each class, followed by workbook activities and project work, and as the study progressed there was little fluctuation from this routine. In Weller's (1991) study of traditional mathematics classrooms, he finds a common daily pattern of instruction: "It was evident that a repeating pattern of instruction occurred which consisted of three distinctive segments: a review, presentation, and study/assistance period" (p. 128). Terry's pattern of instruction did not vary much from Weller's findings.

Any changes in the norms of Terry's mathematics class, such as engagement in the data-analysis think-book activity, were cause for disruption and some uncertainty in the classroom both for the students and for Terry and me. The students were unsure of the expectations as far as the writing assignment was concerned, and Terry and I were constantly grappling with the amount of structure and the type of modeling we could present to the students. Bruner (1986) writes that culture is constantly in the process of being recreated as it is interpreted and renegotiated by its members. In Bruner's view, culture is as much a forum for negotiating and renegotiating meaning and explicating action as it is a set of rules or specifications for action. Even though we knew that

renegotiating some of the norms of Terry's mathematics classroom was reflective of a constructivist approach, we were still uneasy.

The students were an integral part of the classroom culture. Their attitudes towards mathematics and their past experiences had an impact on their perceptions of mathematics and how Terry in turn responded to their perceptions. The students appeared to be used to learning mathematics in a traditional manner, which was reflected in their uncertainties about the writing tasks and their struggles with expressing their mathematical thinking. Their varying personalities and mathematics abilities contributed to life in the classroom. Some of the students needed more assistance than others in mathematics, and this became another factor with respect to the amount of structure that Terry feels is necessary for the classroom to run smoothly. There were days when she felt that she needed a great deal of structure in place, especially the days when she believed that students were struggling with concepts. When Terry saw them struggling with concepts such as area and perimeter, she would go back to a teacher-directed mode of instruction in which she provided the students with definitions.

The students and the work they produced served both as a source of anxiety and as an incentive for Terry and me and to look more carefully at and question what we were doing to facilitate learning during mathematics. Goldsmith and Shifter (1997) state:

> Teachers who are encouraged to listen to and observe students working on mathematical problems, or to explore mathematical problems themselves and reflect on their own processes of understanding, often reconsider their notions of how people learn mathematics. . . . One set of beliefs about learning (often implicitly rather than explicitly held) formed the basis of their past instructional practice and frequently is well entrenched. The new notions that they begin to develop as they reflect on their own and their students' conceptual understandings, however, contradict some of these old assumptions. (pp. 28-29)

The written products that the students produced as a result of the data-analysis project and the school-plan project served as catalysts for Terry's and my reflections on our practice. Our uncertainties with respect to teaching the students reflective thinking in mathematics were a result of our observations and conversations about the students' oral and written explanations.

Time constraints are an ever-present part of the school culture affecting many decisions that Terry has to make. There are always the pressures of playground supervision, staff meetings, and preparation for the next day. The structure of the school day does not allow time for collegial interaction. Administrative support is important to Terry. She said that an administrator has to be knowledgeable about practice that promotes student engagement in mathematics. Her administrator was supportive of an environment that fosters student discourse, but Terry still felt uncertainty about this, wondering if an administrator would view classroom talk as a loss of control on the part of the teacher.

The culture of Terry's school and classroom have a direct bearing on the manner and degree to which she facilitates the process of communication in her classroom. Through the research project we were starting to challenge some of the norms of Terry's mathematics' classroom, and if we had had more time together, these norms might have begun to change. To borrow a quote from Bruner (1986), "The language of education is the language of culture creating, not of knowledge consuming or knowledge acquisition alone" (p. 133). Just as much as the culture of Terry's mathematics class and school may hinder her facilitation of a constructivist approach to instruction, it also is fertile ground for trying new approaches to instruction.

Provincial Tests

The presence of the annual provincial achievement tests was a constant worry for Terry. She worried about getting through the curriculum in time for these tests. The provincial tests, while purportedly aligned with a constructivist philosophy, have been largely unsuccessful in assessing mathematics from a constructivist position. The specter of the provincial tests hindered Terry's willingness to experiment with practice in which the students were engaged in discourse and exploration with each other about mathematical ideas. She did not feel that she had enough time to engage the students in activities conducive to student talk and interaction. Early in the research she told me, "The student textbooks are bright, and they're set up for discussion, which is good, but I panic; I have to get through this and this and this and this" (Interview, Nov. 14, 1998). Terry's delivery of mathematics vocabulary and definitions at the beginning of a new unit of study and her reluctance to take the time to find out what the students already knew about the mathematical concepts were influenced by the provincial tests. She wanted to ensure that the students had the correct conventional mathematics terminology for these tests. Steele (1999) laments the fact that teachers feel they must introduce definitions and mathematics vocabulary without giving students a chance to conceptualize the meanings:

> Teachers should not introduce new words by requiring that students memorize definitions to pass vocabulary tests. [In the words of Vygotsky] Individuals come to learn the meanings of technical terms by transforming them and being transformed by them in the process of internalization. (p. 42)

We did not involve the students in think-book writing tasks after the data-analysis project. Subsequent writing tasks were taken from the textbook and involved the students in writing responses to questions posed. The concern of getting through the mathematics content in readiness for the provincial tests was the reason the textbook tasks were used.

Terry perceived that the data-analysis think-book writing task had taken a great deal of time, and she had many uncertainties about what the students were actually learning by engaging in it.

The provincial tests appear to reflect more transmission than constructivist teaching strategies and therefore reinforce Terry's traditional approach to the interpretation and implementation of the process of communication.

School Subjects and the Structure of the School Day

The structure of the school day in which curriculum subjects are allotted into specific time frames makes it difficult for Terry to implement the goals of constructivist teaching, including the recognition and implementation of language across the curriculum. A timetable conducive to a constructivist philosophy would encourage the students' exploration of subject matter in a multidisciplinary setting. A central theme in current reforms is to present content to students in authentic ways in which literacy, science, and mathematics are brought closer to real world contexts (Ball, 1993; Goodman, 1989; Lampert, 1992). Language across the curriculum practices where talking and writing are recognized as a major part of student learning is easier to implement when there are no strict subject area boundaries. Even with Terry's knowledge of language across the curriculum practices, she found it difficult to implement the process of communication in mathematics. The recommendations of the London group that included Britton (1970), Torbe (1986), and Medway (1980) emphasize the importance of encouraging talking and writing across the curriculum. The Bullock (1975) report, *A Language for Life,* was written in response to the London group's research findings that teachers were not giving students the opportunities for student discourse and

writing. The report emphasizes policies for including reading, writing, and talking across all curriculum areas. These recommendations are still as relevant today, and teachers are still grappling with how to implement them.

Subject area knowledge and confidence are contextual variables important to acknowledge. Shulman (1986) addresses the importance of pedagogical content knowledge to a teacher's ability to make the subject understandable to others. Studies such as those conducted by Cohen (1990), Lampert (1990), and Wineberg and Wilson (1991) indicate that the more knowledgeable a teacher is about a subject matter, the more likely that teacher is to engage the students in opportunities for exploration and discourse. In conversation with Terry about her language arts class, it appeared to me that the students were given many more opportunities for oral discussion and writing than in mathematics. Terry talked about her love of reading and her desire to instill this in her students. This passion was clearly not present in her mathematics teaching. Spillane (2000) and Goldsmith and Shifter (1997) uncover the same findings with respect to their studies of teachers attempting to facilitate practice consistent with the new mathematics reforms. Goldsmith and Shifter (1997) state:

> Many of these teachers are familiar with "process" approaches to teaching other parts of the curriculum such as writing and language arts, and feel that their students would also benefit from mathematics instruction oriented toward their mathematical voices. Yet, because of their own mathematical limitations, they are unable to envision mathematics lessons as anything other than doing activities and working problems from a text. (p. 39)

Terry appeared to be confident about her knowledge of the mathematical concepts she was teaching but there was uncertainty with respect to encouraging the students to talk and write about the process of their mathematical thinking. We were both frustrated with our attempts at coaching the students to talk and write about mathematical processes.

Neither of us had any previous experience in which we facilitated or engaged in talking and writing about mathematical process. Encouraging students to do so was foreign to us. My confidence and knowledge with respect to mathematics was not good, and though I come from a language arts background, in which students are encouraged to write and reflect on their learning, I was not able to transfer this knowledge to mathematics instruction. Goldsmith and Shifter (1997) state that teacher understanding and confidence with mathematics' content influences the degree to which a teacher engages students in exploration:

> If teachers do not have a strong enough grasp of the mathematics they teach, they may not be able to engage their students in an exploration of mathematical ideas beyond calling attention to a variety of possible solution strategies. They themselves may not be able to distinguish valid from invalid reasoning. Teachers who have developed clear and reasoned ideas about the essential aspects of the discipline will be in a far better position to guide students productively through the mathematical terrain. (p. 35)

The impact of the structure of the school day, the delineation of the school subjects, and Terry's subject-area knowledge served to reinforce a transmission model of teaching, inhibiting Terry's interpretation and implementation of the process of communication in mathematics as integral to student construction of meaning.

Differing Philosophical Orientations Towards Constructivism in the Language Arts and Mathematics Fields

The philosophical underpinnings of constructivism are different in language arts and mathematics, and they affect Terry's approaches to instruction. Much of the mathematics literature reflects a Piagetian approach to constructivism that maintains the existence of an external reality to which the learner must gradually accommodate his or her thinking. This standpoint suggests that in the mathematics classroom, learning should

focus on individual knowledge of and about mathematics. The individualism philosophy treats mathematical learning almost exclusively as a process of active individual construction. This position has been exemplified by neo-Piagetian theories that have viewed social interaction as a source of cognitive conflicts that facilitate autonomous cognitive development (Cobb & Bauersfeld, 1995). In the language arts field, constructivism tends to reflect a Vygotskian approach. The Vygotskian stance requires social interaction for the internalization of thought. The source of knowledge is the continuing interactions and reciprocity between a developing child and the world. Curriculum guides and resources reflect the philosophical underpinnings of their subject areas and may affect the decisions a teacher makes with respect to practice reflective of constructivist philosophy.

Not only do the philosophical stances towards constructivism differ in the subjects of language arts and mathematics, but also the term *communication* is described differently. Within the WCP, communication is referred to as the process of communication; and in language arts, communication is not referred to as a process but is considered integral to thinking and learning. At times these differing approaches to the concept of communication created confusion for Terry and me.

The inconsistencies in regard to constructivism and communication constitute an important contextual variable that hindered Terry's implementation of the process of communication in her mathematics teaching.

WCP

Clandinin and Connelly (1995) use the *conduit* metaphor to describe the theoretical knowledge that is packaged for teachers in the form of programs of study and textbooks. The WCP document (Alberta Education, 1996) represented part of the conduit and was a crucial component of this study. The term *constructivism* is not mentioned in the WCP, although it is agreed that the NCTM standards, on which the WCP is based, are nested in a constructivist philosophical position (Cobb, Yackel, & Wood, 1992a; Fennema & Franke, 1992; Heibert, 1999; Schifter & Simon, 1991; Simon, 1995). Simon (1997) explains:

> Constructivism has given us a fundamentally different way to think about knowledge and its development. It has redefined our perception of the human learner and offered us a framework for understanding the complex processes of learning. These changes in our concepts of mathematical knowledge, learner, and learning lead us to question the adequacy of traditional models of teaching. (p. 58)

Terry and I knew that the WCP was based on constructivist philosophy, but it does not describe instructional strategies reflective of this. The Program of Studies only prescribes outcomes, not how to achieve them. It was left to Terry to find teaching ideas from other resources.

There were inconsistencies in the reference to the process of communication in the specific outcomes of the WCP. For example, in the Analyzing Data and Graphs Unit that is part of the Statistics and Probability strand, the process of communication is referred to in all but two of the outcomes. Through facilitating the data-analysis project, Terry and I encouraged talking and writing in the Analyzing Data and Graphs Unit. The students were engaged in conversation with students and teachers in other classrooms and with their peers in their own classroom. The writing in the data-analysis think books was

another way of facilitating the process of communication. In the final three units included in the research study, Measuring and Analyzing Angles, Exploring Geometric Solids, and Measuring Area and Perimeter, all from the Shape and Measurement strand, the process of communication is not listed in any of the specific outcomes. The process of communication should be stated explicitly for all of the outcomes. Talking and writing help students learn mathematical concepts (Britton, 1971; Lampert & Blunk, 1998). Delivery of these outcomes is not possible without the process of communication. This appears to be a critical oversight in the WCP. Despite the fact that the process of communication is not listed in the specific outcomes for the final three units, Terry and I did attempt to provide the students with opportunities for peer talk in the school plan project and the cityscape project. The students were sitting in groups during the creation of their cityscapes, and we tried to encourage interaction through their peer-assessment activity. The students were involved in the process of communication in the writing task associated with the school-plan project in which they had to write about how they used area and perimeter.

The number of outcomes in the WCP also may inhibit a teacher's willingness to take time for student talk and writing. If the WCP is not consistent with respect to promoting the process of communication in all of the specific outcomes, and if a teacher feels overwhelmed by the sheer number of outcomes, a message may be conveyed to teachers that the process of communication is not important in some strands of mathematics and that covering all the concepts is what is important.

Lack of instructional strategies in the WCP reflective of constructivist philosophy, inconsistencies in the inclusion of the process of communication in the specific

outcomes, and the sheer number of outcomes reinforced Terry's transmissionist style of teaching in mathematics.

Textbook

Terry used the mathematics resource *Quest 2000* (Wortzman et al., 1997) exclusively. Like the WCP, the textbook series is also inconsistent in its emphasis on a constructivist stance and its emphasis on the process of communication. *Quest 2000* is a recommended resource because it supports the new reforms in mathematics, and yet the term *constructivism* is not mentioned in the series. However, one of the program principles does reflect a constructivist stance. Program principle #1 states, "Mathematics is a natural activity to children and children learn mathematics best when they construct their own understandings" (p. x). Some teaching ideas that promote the process of communication are included in the teacher's guide, but there appear to be inconsistencies in relation to what is emphasized. There are two pages in the teacher's guide devoted to promoting the importance of writing for learning in mathematics. Included in this explication are writing ideas that reflect a constructivist stance; for example, the use of class and student journals. The same elaboration or importance is not given to the role of talk. The *Quest 2000* series promotes the idea of providing vocabulary definitions at the beginning of a unit of study. In the student textbook there is a section at the beginning of each unit of study called *Words to Know*, where the vocabulary terms and their definitions are listed. The student textbook has many questions that students are expected to answer, but they are not stated in a way that promotes discussion. These activities promote traditional, transmission practice. Weller (1991) argues that authority in the traditional classroom resides with the textbook author. He states, "The expert knowledge

of the teacher was deliberately subjugated to that of the textbook. As a result of that process, the teacher was able to camouflage his role as authoritarian" (p. 133). Terry tended to follow the format of the textbook in her lessons. Terry asked the questions posed in the textbook, and the students answered. The idea for the school-plan project as well as the area and perimeter writing activity were taken from the textbook.

It may have been difficult for Terry to facilitate the process of communication because of her reliance on a textbook that has strategies inconsistent with constructivist views. If teachers do rely extensively on textbooks for instruction, it is crucial that textbooks be reviewed and revised to ensure that the strategies are consistent with the outcomes listed in the Program of Studies.

Terry's Beliefs

Terry's personality and beliefs have a significant bearing on the degree to which she teaches in a manner reflective of constructivist philosophy. She respects her students as knowledge constructors and very much wants to encourage their self-confidence and esteem. She is also concerned with accountability to herself, her students, the parents, and the administration. She is a very conscientious person and, given the pressures of provincial tests, she has found it difficult to venture too far from instructional strategies with which she is already comfortable. Terry believes in constructivism, convinced that opportunities for exploration are important, but she also likes a quiet, structured day. She is aware that her philosophical position on constructivism and her actual practice are not aligned. Mathematics researchers agree that changes in beliefs, knowledge, and practice do not occur in isolation from each other (Fennema, Carpenter, Franke, & Carey, 1992; Hunsaker & Johnston, 1992; Schifter & Fosnot, 1993; Wood, Cobb & Yackel, 1991). It

has been a struggle for Terry to put her idea of constructivism into practice, but this is not unique to Terry. Simon (1997) argues:

> Constructivism does not describe the role of (nor the potential role of) pedagogy in knowledge development. Although it provides an orienting perspective, it does not define a particular teaching practice. Thus, mathematics educators are faced with the challenge of developing new models of teaching that are consistent with constructivist perspectives. (p. 58)

Prawat (1992) finds that even though the mathematics teacher in his study had changed her views of mathematics to become more aligned with the mathematics reforms, her practice still remained very traditional. Terry's teaching tended to stay in a transmission, teacher-directed mode where, in Barnes' (1992) words, "the teacher sees knowledge as existing primarily in a public discipline and will set up classroom communication so that transmission and assessment will predominate" (p. 146). The emphasis on teacher-directed talk is reflective of this position. Terry believes that this format is a good way to get important mathematical concepts across to students. The importance that Terry attributes to the students' acquisition of mathematics vocabulary at the beginning of a unit of study, and her provision of the definitions, are reflective of her desire to stay in control.

Terry's acknowledgement of the importance of constructivism, her desire for a quiet learning environment, and the importance of accountability are critical components of her instructional decisions and the routines that she keeps in place.

Conclusions

Recent reforms within mathematics education have proposed dramatic shifts in philosophical orientation to teaching and learning in mathematics. The WCP document states what students should learn as the result of a unit of study, but it does not state how

a teacher should go about teaching mathematics using a constructivist model. When a curriculum document promotes a new philosophical orientation, it clearly takes time for a teacher to put this change into practice.

Consideration of the contextual variables is crucial to understanding how Terry, a well-educated, informed, and experienced teacher, interprets and implements the process of communication in mathematics. The culture of the classroom and school where Terry and the students engage in routines and accepted protocols, and where time constraints always loom, are integral to the decisions that Terry makes. The landscape outside the classroom, such as the provincial tests, the nature of the school subjects, the subject-specific orientations towards constructivism and the term *communication*, the WCP document, and the *Quest 2000* textbook, have had a great impact on Terry's instructional decisions. Terry's beliefs about constructivism, classroom environments that best facilitate learning, and the need for accountability are also critical factors affecting the degree to which the interpretation and implementation of the process of communication from a constructivist perspective is possible. The contextual variables are significant in creating the tensions and uncertainties that Terry and I shared on a daily basis in our attempts to facilitate talk, writing, and group work. Jones (1997) asserts that it is the tensions created in and between these varying contexts that bring about change:

> What is it that prompts the teacher to consider reconstructing his or her ideas and actions regarding teaching, rather than assimilating the cause of the tension and what is the nature of the supports necessary for change to occur, is a question currently receiving considerable attention. (p. 147)

There was a distinct mathematics culture in Terry's classroom. There were taken-for-granted norms, such as Terry beginning the class with a review of the previous day's work and using the teacher-transmission style of questioning the students. The writing in

the data-analysis project and the increased opportunities for the students to work in group settings required that Terry let go of a practice that was comfortable to her. The talk remained teacher directed most of the time, and the emphasis given to the students' acquisition of vocabulary at the beginning of a unit of study remained unchanged.

Constructivism as a philosophical framework is given different definitions and emphases depending on the source. The *Quest 2000* (Wortzman et al., 1997) textbook, the WCP document (Alberta Education, 1996), and mathematics researchers present varying perspectives on constructivism. Within the specific outcomes of the WCP there are inconsistencies with respect to the emphasis put on the process of communication. Not all specific outcome categories include the process of communication. The *Quest 2000* textbook and teacher's guide have inconsistencies in their approaches to the process of communication. There is more emphasis given to writing than to talking. Two pages of the teacher's guide are devoted to describing writing, whereas there are no descriptors of talk. The textbook includes journal writing ideas for every lesson, but no suggestions for discussion or dialogue.

Terry's belief in constructivism and her desire for a quiet, structured classroom environment are an integral part of her pedagogy, and she struggles with how these variables are reflected in her instructional practice. She knows that her beliefs and practices are often inconsistent. There is still no generally agreed upon consensus in mathematics education as to whether change in belief precedes change in practice or the reverse (Cooney & Shealy, 1997; Thompson, 1992). Terry's reflections on her practice appear to produce many questions regarding a different way of teaching mathematics. Goldsmith and Schifter (1997) argue that it is the dynamic between changing practice and

changing belief that results in the substantial reorganization of teaching: "Without changes in instructional strategies, decisions, and techniques, teachers will be unable to 'walk the walk and talk the talk'" (p. 27). The conversations that occurred between Terry and me during the research formed a foundation upon which we could begin to address this reorganization. Teaching in a constructivist framework requires a fundamental change for teachers in their views of the nature of knowledge (Prawatt, 1992). Examination of one's view on epistemology takes time, and an important place to start is in the context of the classroom with the students.

My presence in the classroom had an impact on the context and culture of the classroom. The discussions Terry and I had about her instructional decisions were possible because of my presence. Had I not been present, there would not have been the opportunities for the rich dialogue that ensued. Pursuing qualitative research and becoming a participant observer result in ongoing tension and uncertainty about how the researcher's presence affects the findings of the study. Another tension involves questions related to the authenticity of the presentation of the research story. Denzin and Lincoln (1998) write:

> The problem of representation will not go away. Indeed, at its heart lies an inner tension, an ongoing dialectic, a contradiction, that will never be resolved. On the one hand there is the concern for validity, or certainty in the text as a form of isomorphism and authenticity. On the other hand there is the sure and certain knowledge that all texts are socially, historically, politically, and culturally located. We, like the texts we write can never be transcendent. (p. 422)

According to Jones (1997), if we understand context to be vitally important to the meaning of a teacher's actions, then we must be concerned with our own poking about at contextual factors.

This study on implementing the process of communication in mathematics portrays the real-life drama of two educators trying to facilitate the new reforms in mathematics, and it reveals the complexities involved. Goldsmith and Shifter (1997) say that images of "reformed" teaching in mathematics vary and that teachers' commitment to changing practice begins and eventually ends with action in the classrooms working with students. This action is not easy; a teacher's orchestration of a curriculum document is a creative endeavor influenced by a multitude of factors. The immediacy of the classroom events are what provides the link between teachers' beliefs and behavior patterns and must be studied in order to gain insight into the nature of the culture of the mathematics classroom (Nickson, 1992). The experiences Terry and I had in working together created an opportunity for examining our beliefs and philosophies about students and mathematics instruction. As the study came to a close we had a feeling that our inquiry had just begun.

The philosophical framework of constructivism was crucial to this study both as a methodological framework for inquiry and as a theoretical framework for understanding how children learn. Interpreting and putting into practice the process of communication evolved through our continuing conversations and deliberations with respect to the instructional strategies Terry utilized and our reflections on how they worked. The study mirrored the essence of constructivism, the constituting of our world through the negotiation with other human beings.

Implications

This research provides another voice with respect to a teacher's interpretation and implementation of the new reforms in mathematics education, in this case specifically the process of communication as described in the WCP document (Alberta Education, 1996). The following implications emerged from this research experience.

1. We can't expect teachers to embrace practice reflective of a constructivist philosophy if the contextual variables do not support this. The contextual factors for the most part supported a transmission style of instruction and inhibited a constructivist stance that would have supported more talk and writing integral to student construction of meaning in mathematics. A close examination of contextual variables is important for any study looking at curriculum implementation. Because a new curriculum document promotes a new way of conceptualizing and implementing its content does not mean that a teacher will change her practice. Many contextual features such as the school and classroom culture, the school subjects and structure of the school day, the WCP, the textbook, and Terry's beliefs impact how she interprets and implements the process of communication.
2. More activities reflective of a constructivist philosophical framework could be included in textbooks and supporting materials. If teachers are relying on textbooks for delivery of instruction, it is important that such texts include teaching strategies that promote student interaction and communication.
3. The specific outcomes of the WCP should be re-addressed to include the process of communication more consistently. Currently, the specific outcomes of the

WCP are inconsistent in their reference to the process of communication. Many omit the process of communication although it is integral to facilitating all of the outcomes.

4. The number of outcomes in the WCP needs to be assessed. There are far more outcomes that can be taught in one school year. The number of outcomes support a transmission style of instruction. Students need time to construct meaning in mathematics. Time is needed to implement teaching strategies reflective of a constructivist approach. Implementing opportunities for student discourse and writing in mathematics cannot be rushed.
5. Opportunities for teachers and other educational stakeholders to have theoretical discussions about the nature of knowledge within the context of classroom practice may be beneficial. Terry's deliberations with respect to constructivism and epistemology were emphasized because of my presence in the classroom.
6. Teachers need to see samples of student writing in mathematics to help them in their implementation of writing in mathematics. Samples of student writing are provided in language arts and social studies curriculum-support documents and are extremely useful as examples for teachers to follow.
7. Teachers need to have opportunities to learn about teaching strategies that support a constructivist approach to mathematics instruction and specifically need to learn strategies to facilitate talking and writing that assist student construction of meaning in mathematics. Opportunities for ongoing professional-development involving mathematics specialists and master teachers are paramount. Stocks and Schofield (1997) write that it is crucial that teachers be active participants in these

opportunities. Occasions for teachers to be paired up with expert mathematics educators in mentoring about teaching situations are vital.

8. Differing interpretations of constructivism in curriculum documents across subject areas can be perplexing for teachers. Opportunities for dialogue among subject area specialists and teachers could be provided so that dialogue about constructivist practice could be held.
9. Change creates uncertainty. It is important that this be acknowledged by those in the mathematics education community and that support and opportunities for open dialogue be present. For example, in this study, implementing the process of communication involved risk taking, and there were many deliberations as a result. Terry and I gave support to each other. It would be beneficial if such supports were in place for all teachers involved in the implementation of curriculum change.
10. It is important that provincial tests be aligned to support a constructivist model of teaching. The pressure of the provincial tests is always present for Terry and is an important factor in how she interprets and implements the process of communication. An example of this is the emphasis she puts on the teaching of vocabulary.
11. Student talk is foundational to students' exploratory writing. It is imperative that mathematics teachers set up opportunities for student talk and interaction. Through talking and writing, students reflect on and clarify their own thinking about mathematical ideas and are able to explain strategies for solving problems.

12. Drawing upon what we know about student writing and talk in other subject areas such as science and social studies would be beneficial to mathematics teachers. It would be helpful if opportunities were created for teacher discussions about language across the curriculum practices that are applicable to mathematics. Revisiting language across the curriculum literature and incorporating it into the Program of Studies and resource materials would benefit all teachers.

Recommendations for Further Research

This study has explored one teacher's interpretation and implementation of the process of communication in mathematics. To highlight the relevance of the study, I offer the following suggestions for research that may further contribute to the ongoing dialogue with respect to teacher implementation of curriculum reform, specifically the process of communication in mathematics.

1. Action research studies exploring effective ways of engaging students in the process of communication in mathematics would be beneficial.
2. Studies that explore more specifically teacher beliefs and knowledge about mathematics and the link to classroom practice would be beneficial.
3. Research literature (e.g., Spillane, 2000) suggests that teachers use subject-specific teaching strategies. Research that explores how teachers might effectively transfer language arts instructional strategies for talking and writing into mathematics would be timely.
4. Studies that examine reform efforts in other subject areas where implementation of instructional techniques reflective of constructivist practice are emphasized might be useful. Successful strategies involving talking and writing as ways of

constructing meaning could be transferred for use in mathematics curriculum implementation.

5. Action-research studies documenting the many different categories of writing that could be used in mathematics, such as think-book writing, journal writing, and dialogue writing, would be beneficial.
6. Further studies in mathematics exploring the link between contexts and teacher change are of paramount importance.

Research Revisited

When I initially generated the question for my research, I did not foresee the enriching journey in which I would engage. My research question seemed straightforward enough, and I believed that Terry and I would have to think about only the activities that would support the students' involvement in the process of communication. I did not envision the many dilemmas and resulting questions that would surface with respect to enacting the process of communication. I gained many insights into the complexities of carrying out this process in a mathematics class; more important, I became aware of the many variables that come into play when making instructional decisions.

As a teacher I had never been comfortable with mathematics, and although I had many experiences engaging my students in student talk and reflective writing in social studies and science, I did not transfer these experiences into my teaching of mathematics. I was unsure of how to integrate talking and writing into mathematics and sometimes believed that I should have had more expertise with this if I was going to engage in a research study that explored the process of communication in mathematics. Mathematics always troubled me both as a student and as a teacher. Exploring the process of

communication within Grade 6 mathematics proved to be a challenge for both Terry and me, intellectually and practically. Terry's knowledge of mathematical concepts impressed me; she had a confidence with mathematics that filtered into her lessons. I did not have this knowledge and confidence and sometimes felt at a loss as to what to suggest for instructional strategies. We both thought of ourselves as constructivist teachers but had difficulty engaging the students in classroom activities that reflected our beliefs.

Terry's willingness to engage in the research is something for which I will be forever grateful. She always offered thoughtful responses to my questions and was not afraid to admit emotions of frustration or uncertainty. There was never a moment when she wanted to disengage from the study, and even in stressful times such as student report card periods, she was always available to talk about the research.

The students always welcomed me into their class. In the beginning they wondered who I was and what I was doing, but they did not take long to accept me and treat me as another adult within the classroom. They seemed to enjoy their one-on-one conversations with me and liked that little bit of extra attention that is so difficult for students and teachers alike to attain.

Bauersfeld (1995) refers to the "language game" within the mathematics classroom in which we gain access to the subculture by active participation. Terry's classroom represented such a subculture. Distinct norms were present from the beginning with respect to how communication was used. The students knew when they were supposed to talk and how they were supposed to respond to the teacher-directed questions. They knew the protocol for using vocabulary terminology and wrote according to the structure modeled for them. Norms take a long time to change.

This study reflects the complexities inherent in our understanding of a classroom culture. Bauersfeld (1995) asks the question, "How do teacher and students arrive at taken-as-shared issues, such as activities, objects, language games, and social regularities?" (p. 281). The answer is found in breadth of inquiry that looks at all the possibilities: the students as people with distinct histories, the teacher with her distinct history, the researcher with her distinct history, and the many other contextual variables.

Terry challenged herself to honestly look at her beliefs and practice, and even though much of the instruction was teacher directed, she was attempting to change this.

This study challenges all of us to reflect honestly on our own beliefs about knowledge and teaching and how this is transformed into practice. The study elicits more questions than it provides answers, but that is what it means to be a lifelong learner and constructor of meaning.

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APPENDIX A

NCTM COMMUNICATION STANDARDS

NCTM Communication Standards

K-4	5-8	9-12
The study of mathematics should include numerous opportunities for communication so that students can:	*The study of mathematics should include opportunities to communicate so that students can:*	*The mathematics curriculum should include the continued development of language and symbolism to communicate mathematical ideas so that all students can:*
• relate physical materials, pictures, and diagrams to mathematical ideas • reflect on and clarify their thinking about mathematical ideas and situations • relate their everyday language to mathematical language and symbols • realize that representing, discussing, reading, writing, and listening to mathematics are a vital part of learning and using mathematics.	• model situations using oral, written, concrete, pictorial, graphical, and algebraic methods • reflect on and clarify their own thinking about mathematical ideas and situations • develop common understandings of mathematical ideas, including the role of definitions • use the skills of reading, listening, and viewing to interpret and evaluate mathematical ideas • discuss mathematical ideas and make conjectures and convincing arguments • appreciate the value of mathematical notation and its role in the development of mathematical ideas.	• reflect upon and clarify their thinking about mathematical ideas and relationships • formulate mathematical definitions and express generalizations discovered through investigations • express mathematical ideas orally and in writing • read written presentations of mathematics with understanding • ask clarifying and extending questions related to mathematics they have read or heard about • appreciate the economy, power, and elegance of mathematical notation and its role in the development of mathematical ideas
(NCTM, p. 26)	(NCTM, p. 78)	(NCTM, p. 140)

APPENDIX B

MATHEMATICAL PROCESSES

Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and to encourage lifelong learning in mathematics. Students are expected to:

• *Communication*	• communicate mathematically
• *Connections*	• connect mathematical ideas to other concepts in mathematics, to everyday experiences and to other disciplines
• *Estimation and Mental Mathematics*	• use estimation and mental mathematics where appropriate
• *Problem Solving*	• relate and apply new mathematical knowledge through problem solving
• *Reasoning*	• reason and justify their thinking
• *Technology*	• select and use appropriate technologies as tools to solve problems
• *Visualization*	• use visualization to assist in processing information, making connections and solving problems.

The Common Framework incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning.

APPENDIX C

PROGRAM PRINCIPLES FROM *QUEST 2000*

Program Principles from *Quest 2000*

1	Mathematics is a natural activity to children, and children learn mathematics best when they construct their own understandings.
2	Children learn mathematics through worthwhile activities as they explore, communicate, and reflect on important mathematical ideas and procedures in a variety of contexts.
3	The teacher's role is to facilitate student thinking, work, and reflection. Questions rather than telling should be the focus of teaching.
4	Mathematics learning is best achieved through posing and solving problems.
5	Children learn mathematical skills through problem solving, and practise these skills through independent work.
6	Mathematics should not be taught in isolation from other activities or from the real world. This being so, parents and teachers can become partners in fostering eager learners.
7	Communication is core to all mathematics activity. Communication is promoted through groups working cooperatively and is enhanced by children writing about their mathematical ideas, processes, and questions.
8	Mathematics is accessible to children of different experiences, interests, language, learning styles, and abilities.
9	The tools of technology—computers and calculators—and concrete objects help children learn to solve problems in ways that will prepare them for the future.
10	Assessment should be broad-based. It should focus on ongoing observations and the work students do over a period of time, and not just on the answers and solutions to specific questions and problems.

APPENDIX D

TEACHER CONSENT FORM

Teacher Consent Form

Name:

Address:

I _____ consent to be a participant in Monica Ellis's research study on one teacher's interpretation of the "communication standards" of the *Alberta Program of Studies for K-9 Mathematics Western Canadian Protocol for Collaboration in Basic Education* curriculum under the following conditions:

1. The nature and purpose of the study will be explained to me.
2. Pseudonyms will be used in the study.
3. The study is voluntary and I have the right to withdraw at any time without penalty.
4. The data will be used to write a PhD dissertation. The data may be used for conference presentations and journal articles.
5. The type of data to be collected will be in the form of tape-recorded interviews and written field notes and may include the collection of some documents such as lesson plans with my permission.
6. Any information that identifies the teacher will be destroyed upon completion of the research.

Signature _____ Date_____

Monica Ellis may be contacted at 780-492-4273, Ext. 255, c/o Department of Elementary Education, 551 Education South, Edmonton, AB, T6G 2G5

APPENDIX E

STUDENT CONSENT LETTER

Student Consent Letter

Dear Parent or Guardian:

I am conducting a study in_____ 's classroom on the topic of how a teacher interprets the "communication standard" of the new mathematics curriculum. This study is being done in partial fulfillment of the requirements for a PhD in Education. My focus is on the teacher's interpretation of the "communication standard," but from time to time I would like to copy some of the students' work samples as a source of discussion for the teacher and myself about the teacher's practice and as possible examples for inclusion in the dissertation document.

There will be complete anonymity for the students, teacher, and school. Pseudonyms will be used. Data collected for this study will be used to write a PhD dissertation. The data may also be used for conference presentations and journal articles.

The use of the students' written work is completely voluntary and the right to withdraw this consent can happen at any time without penalty.

If you have any questions or concerns you may contact me, Monica Ellis at 403-436-1195.

Sincerely,

Monica Ellis

I _____ hereby give permission for_____ work samples to
(signature of parent or guardian) (student's name)
be used from time to time in Monica Ellis's PhD study.

_____
(signature of student)